T. Y. B. Sc. Electronic Science

Paper IV: Semester IV

Foundation of Nanoelectronics

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(SPEED)

Acknowledgment

Major problem faced by T. Y. B.Sc. students is availability of text books and most of them are reference books. Sometimes books may not be available for the students. Hence SPEED (Society for Promotion of excellence in electronics discipline) has taken a step forward to help these students. SPEED has contributed by online publishing reference notes for the "Foundation of Nanotechnology" for T. Y. B.Sc. Electronic science students. These notes will really be helping them for understanding the subject as well as preparation for examination. We are thankful to Dr. S. N. Khan, Associate professor, Pemraj Sarda college,Ahmednagar for his major contribution in preparation of notes. Such contribution from staff members is really appreciable.

Dr. A. D. Shaligram

Chairman, SPEED

Paper IV: Semester IV

EL -344: Foundation of Nanoelectronics

Learning Objectives:

1. To learn essential principles of Electromagnetics
2. To know the principles of quantum mechanical aspects
3. To study the basics of nanoelectronics.

Unit 1: Essential Electromagnetics [14]

Lorentz force-Motion of charged particle in E-M fields, cyclotron frequency, Hall effect, Maxwell's equations, Relation with laws of Electrodynamics, Equation of continuity, Poynting vector theorem, Wave equation for E and H, properties of EM waves in conducting and nonconducting media , Skin depth.

Unit 2: Quantum mechanical aspects [12]

Particles and Waves: Classical particles, Light as wave and particle, Wave particle duality and Uncertainty principle, Wave mechanics: The Schrödinger wave equation, wave mechanics of particles, Infinite potential well, Qualitative treatment of square wave potential with special reference to tunneling phenomenon, atoms and atomic orbital.

Unit 3: Statistical aspects [10]

Classical statistics, Gaussian distribution, Poisson distribution, Fermi-Dirac, Bose Einstein, Maxwell Boltzmann statistics, Time and length scales of the electrons in solids, statistics of electrons in solids and nanostructures, Density of states of electrons, electron transport, Conductivity of metals.

4. Nanoelectronics [12]

Importance of nanoelectronics, Top down approach, Bottom up approach, Lithography, Nanostructure devices like resonant-tunneling diode, electrons in quantum wells, electrons in quantum wire, electrons in quantum dots, Quantum dot applications, Flash Memory.

Recommended Books:

1. George W. Hanson "Fundamentals of nanoelectronics", LPE, Pearson Education
   V. Mitin ,Viatcheslav A. Kochelap , Michael A. Stroscio Vladimir
2. "Introduction to NanoelectronicsScience ,nanotechnology , Engineering and Applications" Cambridge University Press 2008
3. Ben G. Streetman ,Sanjaykumar Banerjee "Solid State Electronic Devices ", 6th Edition
4. Kraus and Fleisch "Electromagnetics with applications" McGraw Hill, 5th edition
5. Electromagnetics by B.B. Laud, Wiley Edition
6. Donald A.Neaman, "Semiconductor Physics and devices" 3rd edition TMH

Khan S.N. Sarda College Ahmednagar.

Ch. II & IV Introduction to Nanoelectronics

* Nanoelectronics promises to improve, amplify & partially substitute for the well-known field of microelectronics. The prefix micro denotes the one millionth & used to indicate smallest features of a conventional electronic device having length scales approximately a micrometer.

The prefix nano denotes one billionth. Thus, in nanoelectronics the dimensions of the devices should be thousand times smaller than those of microelectronics. Atoms, DNA, proteins, viruses & transistors are all broadly classified as nanoscale objects. (within the factor of 100 of a nanometer).

* Top - Down Approach :-

Top-down approach refers to slicing or successive cutting of bulk material to get nano size particles.

e.g. Milling.

The biggest problem with top down approach is imperfection of surface structure, significant crystallographic damage to the process pattern. This imperfection leads to extra challenges in the device design & fabrication. But this approach leads to the bulk production of nanomaterials.

As the feature size is reduced towards a nanometer, more & more purely quantum effects begins to emerge.

e.g. when the gate oxide thickness of a metal-oxide-semiconductor FET (MOSFET) goes below 1 - 2nm, significant tunneling through the gate oxide occurs.

The disadvantage is that as the feature size is reduced, costs increases.

* **Bottom-up approach :-**

Bottom-up approach refers to the build up material from the bottom atom by atom, molecule by molecule or cluster by cluster.

e.g. Colloidal dispersion in synthesis of nanoparticles.

Bottom up approach promises for better chance to obtain nanostructure with less defect, & more homogeneous.

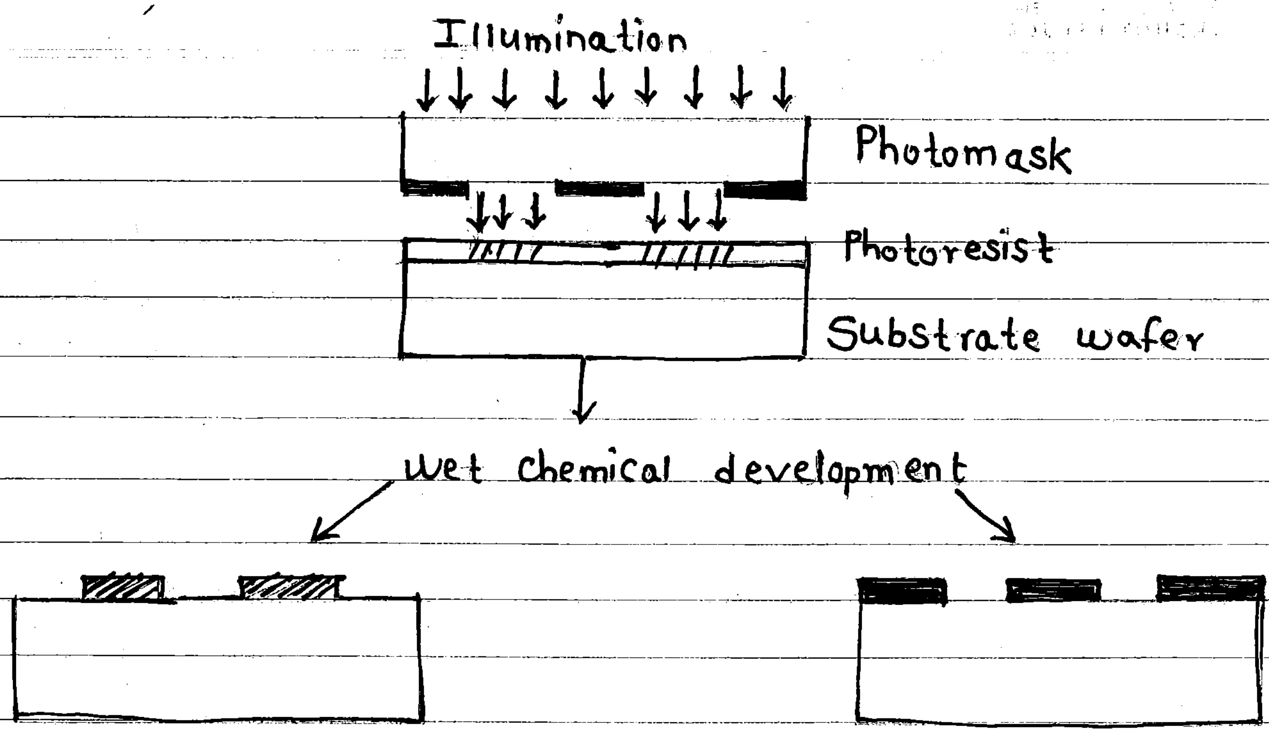
* **Lithography :-**

Lithography is the process of using electromagnetic energy to transfer a pattern from a mask to resist layer deposited on the surface of a substrate (called as wafer) to form an electrical circuit.

Lithography Process Steps :-

1] A photosensitive emulsion called a photoresist is applied to the wafer (mostly silicon wafer).

2] Optical energy (light) is directed at a photomask containing opaque (non-transparent) & transparent regions that correspond to the desired pattern. The light that passes through the photomask reaches the wafer, illuminating the desired pattern on the resist.



Negative Photoresist

Photoresist is photopolymerised where exposed & rendered insoluble to the developer solution.

Positive Photoresist

Developer Solution only dissolves photoresist in the exposed areas.

Fig:- 1 to 3 steps of the Lithography Process.

In Projection lithography, lenses are used to focus the light before & after the photomask.

3] Sections of the photoresist that are exposed to the light coming through the mask undergo chemical reactions

a] For a negative photoresist, the resist material is initially soluble & through a chemical reaction when exposed to light, becomes insoluble. When the wafer is later washed with a solvent, the areas that were unexposed (i.e. where the photomask blocked illumination) dissolve, & the exposed areas of the photomask remain.

b] In a positive photoresist, the resist material is initially insoluble, & through chemical reaction when exposed to light, becomes soluble. when the wafer washed with a solvent, the areas that were exposed to the illumination dissolve, & the unexposed areas remain.

4] Following steps may be performed to transfer the pattern from the resist to the wafer.
For example,

a] Etching may be used to remove substrate material. The photoresist resists the etching & protect section of the wafer that it covers. After etching the resist is removed, leaving the desired structure.

b] Material may be deposited, e.g. metallization onto the wafer. Then the photoresist can be removed, leaving the deposited material in areas that were not covered by the resist.

c] Doping can occur, e.g. a beam of dopant ions can be accelerated towards the wafer. The resist blocks the ions from reaching those regions of the wafer covered by the resist & thus creates regions of doping in areas not covered by the resist. This is known as ion implantation.

- Importance of nanoelectronics :-

1] Consumers have become used to electronic products becoming simultaneously smaller & cheaper & yet more powerful.

2] This trend is the desire for Companies to be Competitive & reflects the broad wish of Consumers for Smaller, faster, cheaper & better electronic products.

3] The reduction in product size is due to reducing the size of indivisual Components like transistors. This leads to improved functionality, as more devices can be packed into a given area.

4] The economic advantages of Small device size since the Cost of integrated circuit chips is related to the number of chips that can be produced per silicon wafer. ∴ Higher device density leads to more chips per wafer & reduced Cost.

5] Includes possibility of ultrasmall, low-power electronic products, Such as Communication & Computing devices & embedded sensors.

Ch. II

* Particles & Waves :-

- classical particles :-

A particle can be characterized by the momentum Vector p & the kinetic energy E that depends on the momentum. The change of momentum with time is defined by Newton's 2nd law :

$$\frac{dp}{dt} = f \qquad \text{— (1)}$$

where, t = time & f = external force.

If the force is absent, then $dp/dt = 0$ i.e.

p = Constant. This is Called as momentum Conservation law Valid for a mechanical system in the absense of external forces.

In classical mechanics, we assume that any particle has a very small size in comparison with the space where the particle is located. Such a particle is called as point particle.

The co-ordinate vector r of a point particle & the particle velocity v are related by,

$$\frac{dr}{dt} = v \quad \text{———} \quad (2)$$

To obtain the relationship among the velocity v & momentum p & the energy of a particle E, calculate the power associated with the force F.

Multiply the eq^n (1) on both sides by v

$$\therefore \quad v\frac{dp}{dt} = fv.$$

fv is equal to the rate of energy change dE/dt.

$$\therefore \quad \frac{dE}{dt} = fv$$

$$\therefore \quad \frac{dE}{dt} = v\frac{dp}{dt} \quad \text{———} \quad (3)$$

$$\therefore \quad v = \frac{dE}{\cancel{dt}} \times \frac{\cancel{dt}}{dp}$$

$$\therefore \quad v = \frac{dE}{dp} \quad \text{———} \quad (4)$$

$\therefore$ Derivative w.r.t. the vector p also gives the vector v with components

$$v_x = \frac{dE}{dp_x}, \quad v_y = \frac{dE}{dp_y} \quad \& \quad v_z = \frac{dE}{dp_z}$$

Now consider an important case that particle is moving in a potential field.

The force is defined as derivative of a potential $V(r)$ with respect to particle coordinate

$$f = \frac{-dV}{dr}$$

The vector operator $\frac{d}{dr} \equiv \left\{ \frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz} \right\}$

& noted as, $\frac{d}{dr} \equiv \nabla$ (grad).

$\therefore$ $\frac{dV}{dr}$ is called gradient of function $V(r)$ i.e. $\nabla V(r)$.

$\therefore$ from eq^n (2),

$$v \frac{dp}{dt} + \frac{dV}{dr} \frac{dr}{dt}$$

$$= \frac{d}{dt}\left(E + V(r)\right) = 0$$

$\therefore$ The value of the kinetic energy plus potential energy

$$\boxed{H \equiv E + V(r)} \quad \text{—— (5)}$$

represents total energy of the particle.

The total energy of a particle in a potential field does not change during its motion.

$\therefore$ From the law of energy conservation, $\frac{dH}{dt} = 0$.

When H is considered as a function of two variables p & r, it is called the Hamiltonian function or Hamiltonian.

A point particle moving in free space may be characterised by a mass m & by the K.E.

$$E = \frac{p^2}{2m} \quad \text{—— (6)}$$

$$\therefore \quad \boxed{H \equiv \frac{p^2}{2m} + V(r)}$$

— Classical waves :-

Classical waves include sound waves in air, sea waves, & elastic waves in solids, electromagnetic waves, & gravitational waves. In classical physics, wave motion arises in extended continuous media with an interaction between nearest elements of the medium. Such interaction gives rise to the transfer of a distortion from one element to another & to a propagation of this distortion through the medium.

Consider a model of one-dimensional medium, elements of which are represented by "atoms" connected by massless springs. Vibrations in such a linear atomic chain are governed by the laws of classical mechanics. If the chain is infinitely long, let the equillibrium distance between atoms be a. Thus, the equillibrium position of the n^{th} atom is $z_n = na$, & the displacement of this atom from its position is denoted by u_n.

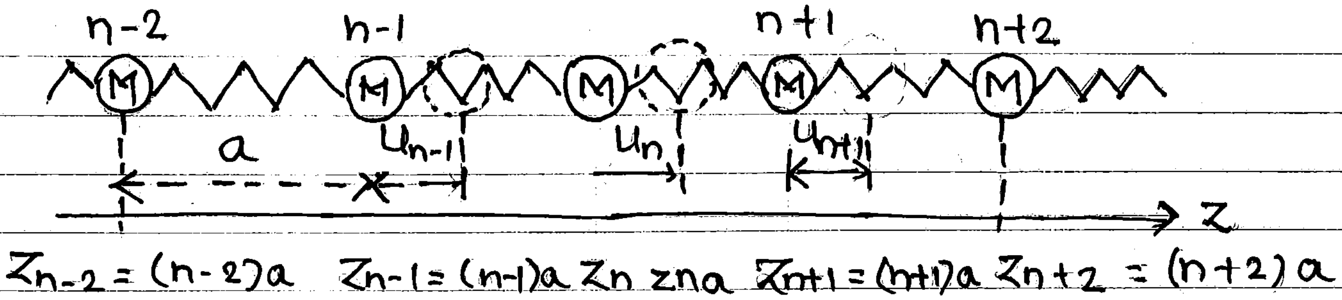


Fig :- of identical atoms of mass M. The springs represent interatomic forces i.e. interaction between nearest elements of the medium.

If the displacements of atoms from their equillibrium position are not too large, then

* $$f = -\beta u \quad \text{--- (1)}$$

where, u = change of spring length.
β = spring Constant
f = force exerted by the string.

The total force f_n acting on the n^{th} atom Coupled with its two nearest neighbors by two Springs as,

$$f_n = -\beta(u_n - u_{n+1}) - \beta(u_n - u_{n-1}). \quad \text{--- (2)}$$

Hence the Newton equation of motion for the n^{th} atom is,

$$M\frac{d^2u_n}{dt^2} = -\beta(2u_n - u_{n+1} - u_{n-1}) \quad \text{--- (3)}$$

This set of linear differential eq^{n}s, eq^n (3) describes wave-like processes.

* Uncertainty Principle :-

Heisenberg's uncertainty Principle :-

Statement :- The principle states that one Cannot measure the position Coordinate & Corresponding momentum of the particle Simultaneously with arbitrar accuracy.

If Δx & Δp_x are the uncertainty in the simultaneous measurement of x- Coordinate & the Corresponding momentum, then according to Heisenberg's uncertainty principle, the product of uncertainties is always greater than or equal to Planck's Constant (h).

$$\therefore \Delta x . \Delta p_x \geq h \quad \text{--- (i)}$$

Similarly, $\Delta y . \Delta p_y \geq h$ --- (ii)

& $\Delta z . \Delta p_z \geq h$ --- (iii)

$\Big\}$ --- (1)

The principle implies that if one tries to determine position coordinate (x) more & more accurately i.e. $\Delta x \rightarrow 0$, then the momentum becomes more & more uncertain i.e. $\Delta p_x \longrightarrow \infty$ & vice versa.

one can never determine both x & p_x as accurately as one wishes i.e. we can never have $\Delta x \rightarrow 0$ & $\Delta p_x \rightarrow 0$ simultaneously. Thus the uncertainty product $\Delta x . \Delta p_x$ can never be made less than h.

Precise mathematical statement of the principle, states that,

$$\Delta x . \Delta p_x \geqslant \frac{\hbar}{2}$$

$$\Delta y . \Delta p_y \geqslant \frac{\hbar}{2}$$

& $$\Delta z . \Delta p_z \geqslant \frac{\hbar}{2}$$

where, $\hbar = \dfrac{h}{2\pi}$ called modified Plank's Const.

The 2nd statement of the uncertainty principle is that it is impossible to simultaneously describe with absolute accuracy the energy of a particle & the instant of time the particle has this energy. If the uncertainty in energy is given by ΔE & the uncertainty in the time by Δt, then the uncertainty principle is described as,

$$\Delta E . \Delta t \geqslant h$$

where h = Planck's Constant.

* ~~Wave Particle~~

— Illustrative Examples :-

1] A 10 gm bullet shoots through a cylindrical tunnel of 5 cm diameter. What would be the uncertainty in the velocity of bullet?

→ Given :- $\Delta x = 5\text{ cm} = 5 \times 10^{-2}\text{ m}$

$m = 10\text{ gm} = 10 \times 10^{-3}\text{ kg}$

Now,

$$\Delta x . \Delta p_x \approx h$$

$$\therefore \Delta x . m \Delta V_x \approx h$$

$$\therefore \Delta V_x \approx \frac{h}{m \Delta x}$$

$$= \frac{6.6 \times 10^{-34}}{10 \times 10^{-3} \times 5 \times 10^{-2}}$$

$$= \frac{6.6}{5} \times 10^{-34+4}$$

$$\therefore \boxed{\Delta V_x = 1.3 \times 10^{-30}\text{ m/s}}$$

The error in V_x is too small to be measured experimentally. $\therefore$ The error is as good as zero.

2] Suppose the instaneous position of 1 gm particle is measured within maximum possible error of 10^{-3} cm. What is the error in the corresponding velocity?

→ Given :- $m = 1\text{ gm} = 10^{-3}\text{ kg}.$

$\Delta x = 10^{-3}\text{ cm} = 10^{-5}\text{ m}.$

Now, $\Delta x . \Delta p_x \approx h$

$$\therefore \Delta x . m \Delta V_x \approx h$$

$$\therefore \Delta V_x \approx \frac{h}{m \Delta x}$$

$$\therefore \Delta V_x = \frac{6.6 \times 10^{-34}}{10^{-3} \times 10^{-5}}$$

$\therefore \boxed{\Delta V_x = 6.6 \times 10^{-26} \text{ m/s.}}$

The error in velocity is too small to be detected experimentally.

3] The $e^{\ominus}$s are allowed to pass through a crystal with lattice constant 1 A°. What is the uncertainty in its velocity?

→ Given :- $\Delta x = 1 A^\circ = 10^{-10}$ m.

Now,
$$\Delta V_x \approx \frac{h}{m \Delta x}$$
$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}}$$
$$\boxed{\Delta V_x = 7.3 \times 10^{6} \text{ m/s.}}$$

If $\Delta V_x = 7.3 \times 10^{6}$ m/s, then the velocity V_x itself must be at least of this order.

4] Estimate the size of the atom using uncertainty principle.

→ Experimental observations show that the energy of $e^{\ominus}$ in an atom is few electron-volts (eV).

Let us take this energy to be $E = 10$ eV.

$$\therefore \quad E = 10 \times 1.6 \times 10^{-19} \text{ J} \quad (1 \text{eV} = 1.6 \times 10^{-19} \text{J}).$$
$$= 1.6 \times 10^{-18} \text{ J}.$$

Now,
$$E = \frac{p^2}{2m}$$
$$\therefore \quad p = \sqrt{2mE}$$

The $e^{\ominus}$ may have forward or backward momentum along the given direction.

$$\therefore \Delta p \approx p - (-p) = 2p.$$
$$\therefore \Delta p \approx p = \sqrt{2mE}$$
Now,
$$\Delta x \approx \frac{h}{\Delta p} = \frac{h}{\sqrt{2mE}}$$
$$\therefore \Delta x = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^{-18} \times 1.6}}$$
$$\boxed{\Delta x = 3.87 \times 10^{-10} \text{ m}}$$
Thus, the diameter of the atom is about 10^{-10} m. This is in good agreement with experimental measurements of atomic diameter.

5] Prove that − $\Delta E \cdot \Delta t \geq h$

→ Proof :- we have,
$$\Delta x \cdot \Delta p_x \approx h$$
Now, $E = \frac{p^2}{2m}$ for motion along X-axis.
$$\therefore \Delta E = \frac{2p_x \cdot \Delta p_x}{2m}$$
$$= \frac{mV_x \cdot \Delta p_x}{m}$$
$$\Delta E = V_x \cdot \Delta p_x$$
$$\therefore \Delta E \cdot \Delta t = V_x \cdot \Delta p_x \cdot \Delta t$$
But $V_x \cdot \Delta t = \Delta x$
$$\therefore \Delta E \cdot \Delta t = \Delta x \cdot \Delta p_x \approx h$$
$$\therefore \Delta E \cdot \Delta t \approx h$$
In general, $\Delta E \cdot \Delta t \geq h$

6] Calculate uncertainty principle in position of $e^{\ominus}$ moving with velocity 10^5 m/s.

→ Given :- $\Delta V_x = 10^5$ m/s.

Now,

$$\Delta x \cdot \Delta p_x \approx h$$

$$\therefore \Delta x \cdot m \Delta V_x \approx h$$

$$\therefore \Delta x = \frac{h}{m \Delta V_x}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^5}$$

$$= 0.725 \times 10^{-34+26}$$

$$\boxed{\Delta x = 0.725 \times 10^{-8} \text{ m}}$$

7] Determine the Current density in Copper wire of diameter 1 mm carrying a current of 3.142 A. What is the smallest possible uncertainty in position of an $e^{\ominus}$ moving with velocity 10^6 m/s.

→ i] To Calculate Current density of Cu wire :-

Current density $(J) = \dfrac{I}{A}$

Given :- $d = 1\text{ mm} = 10^{-3}$ m. $\therefore r = 0.5 \times 10^{-3}$ m

$I = 3.142$ A.

$\therefore$ Area $(A) = \pi r^2$

$= 3.142 \times (0.5 \times 10^{-3})^2$

$\therefore$ Current density $(J) = \dfrac{I}{A}$

$$= \frac{\cancel{3.142}}{\cancel{3.142} \times (0.5 \times 10^{-3})^2}$$

$$E = \frac{n\pi^2\hbar^2}{2m^2}$$

$$= \frac{1}{0.25 \times 10^{-6}}$$

$$= \frac{10^6}{0.25} = 4 \times 10^6$$

$$\therefore \boxed{J = 4 \times 10^6 \ A/m^2}$$

ii] To Calculate uncertainty in position of an $e^{\ominus}$:-

Given :- $\Delta V_x = 10^6$ m/s.

Now, $\Delta x \cdot \Delta p_x \approx h$

$\therefore \Delta x \cdot m \Delta V_x \approx h$

$$\therefore \Delta x \approx \frac{h}{m \Delta V_x}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6}$$

$$= 0.725 \times 10^{-34+25}$$

$$= 0.725 \times 10^{-9} \ m$$

$$\boxed{\Delta x = 0.725 \ nm}$$

8] Calculate the ground state energy of an $e^{\ominus}$ confined to move freely between two ends separated by (10 A°). ($m = 9.11 \times 10^{-31}$ kg).

Given :- $m = 9.11 \times 10^{-31}$ kg.

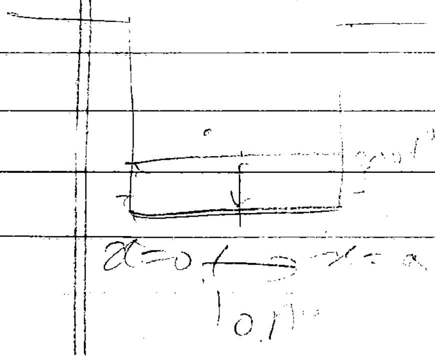
$a = 10A° = 10 \times 10^{-10}$ m. $= 10^{-9}$ m

The ground state energy of $e^{\ominus}$ is given by,

$$\therefore E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$$

$\because \hbar = \frac{h}{2\pi}$

$\therefore \hbar^2 = \frac{h^2}{4\pi^2}$



$$= \frac{(3.142)^2 \times \frac{h^2}{4\pi^2}}{2ma^2}$$

$$= \frac{h^2}{8ma^2} = \frac{(6.6\times10^{-34})^2}{8\times9.11\times10^{-31}\times(10^{-9})^2}$$

$$= \frac{43.56\times10^{-68}}{72.88\times10^{-31-18}} = 0.5976\times10^{-68+18}$$

$$\boxed{E_1 = 0.5976\times10^{-50}\ J}$$

9]. An $e^{\ominus}$ has speed of 6000 m/s with an accuracy of 0.05%. Calculate uncertainty with which the position of $e^{\ominus}$ can be located.

→ Velocity of $e^{\ominus}$ is,

$$\Delta V_x = 6000\ m/s \text{ with accuracy of } 0.05\%$$

$$\therefore \Delta V_x = 60\cancel{0}\cancel{0} \times \frac{0.05}{1\cancel{0}\cancel{0}}$$

$$= 3.00\ m/s$$

$$\therefore \boxed{\Delta V_x = 3\ m/s}$$

Now,

$$\Delta x \cdot \Delta p_x \approx h$$

$$\therefore \Delta x \cdot m\Delta V_x \approx h$$

$$\therefore \Delta x \approx \frac{h}{m\Delta V_x}$$

$$= \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times3}$$

$$= \frac{2.2}{9.1}\times10^{-34+31}$$

$$= 0.2417\times10^{-3}\ m$$

$$\boxed{\Delta x = 0.2417\ mm}$$

10] An $e^{\ominus}$ has speed of 2×10^{4} cm/sec accurate to 0.01%. What is uncertainty in the position of $e^{\ominus}$.

Velocity of $e^{\ominus}$ is,

$\Delta V_x = 2 \times 10^4 \ cm/sec = 2 \times 10^4 \times 10^{-2} \ m/s = 2 \times 10^2 m$,

with accuracy of 0.01 %.

$$\therefore \Delta V_x = 2 \times \cancel{10^2} \times \frac{0.01}{\cancel{100}}$$

$$= 2 \times 0.01$$

$$\boxed{\Delta V_x = 0.02 \ m/s.}$$

Now,

$$\Delta x \cdot \Delta p_x \approx h$$

$$\therefore \Delta x \cdot m \Delta V_x \approx h$$

$$\therefore \Delta x = \frac{h}{m \Delta V_x}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.02}$$

$$= \frac{6.6}{0.182} \times 10^{-34+31}$$

$$= 36.26 \times 10^{-3} \ m$$

$$\boxed{\Delta x = 36.26 \ mm}$$

* Photoelectric effect :- When light is made incident on a metal surface, $e^{\ominus}s$ are emitted from the surface. This effect is called photoelectric effect & the $e^{\ominus}s$ emitted are called photoelectrons.

If these $e^{\ominus}s$ are allowed to flow in a closed circuit the current obtained is called photoelectric current. Some of the experimental observations about the photoelectric effect are :-

a] Photoelectric Current increases with increase in the intensity of incident light.

b] Photoelectric effect is not observed if the frequency of light (ν) is less than certain frequency (ν_0), no matter how intense is the ~~light~~ incident beam. The freq^n ν_0 is called threshold freq^n. Its value depends upon the nature of the emitting surface.

c] There is almost no time-lag between the incidence of light & emission of photoelectrons. i.e. the photoelectric effect is observed instantaneously.

d] The photo$e^{\ominus}$s have different kinetic energies in the range 0 to E_{max}. Max^m kinetic energy of photo-$e^{\ominus}$s is proportional to the frequency of light & is independent of intensity of incident light.

Wave theory does not predict the existence of threshold freq^n. However, emitting surfaces of different materials possesses different threshold freq^{n}s. Thus, classical wave theory of light fails to explain photoelectric effect satisfactorily.

* <u>Wave - particle duality :-</u>

<u>Dual nature of light :-</u>

The electromagnetic radiation such as light has two aspects, a wave aspect & a particle aspect. Interference, diffraction & polarization of light can be explained if the radiation is assumed to consist of waves having frequency ν & wavelength λ.

On the other hand, photoelectric effect or Compton effect can be explained only if it is assumed that radiation Consists of particles Called photons with energy E & momentum p.

Thus electromagnetic radiation has dual nature, waves & particles, called wave particle duality of electromagnetic radiation.

Let us Consider dual nature of matter. we will see that moving particles like $e^{\ominus}s$ or protons exhibit the wave particle duality.

- **De-Broglie's Hypothesis :- Matter waves :-**

Waves & particles are only the two modes of energy propagation in nature. Like radiation, matter should also exhibit wave particle duality. That is matter (particles) should exhibit wave like behavior under Certain proper Condition.

On the basis of this argument, Louis De Broglie (1924) put forward the hypothesis

"Matter Considered to made up of discrete particles such as atoms & molecules, these particles Can exhibit wave like behavior under proper Conditions".

- **De Broglie's equation :-**

The radiation is not being emitted in Continuous fashion, but in discrete bundles of energy. These bundle or packets of radiant energy are termed as quanta or photons.

The energy of photon is given as,

$$E = h\nu \qquad \text{——} \quad (1)$$

where, h = Planck's Constant

ν = freq^n of radiation.

According to mass-energy relation of Einstein, the energy is given by,

$$E = mc^2 \quad \text{——} \quad (2)$$

where, m = mass of the photon
c = Velocity of light

from eq^{n}s (1) & (2) we have,

$$mc^2 = h\nu$$

$$\therefore \quad m = \frac{h\nu}{c^2}$$

But since the photon is always in motion & velocity of photon is always equal to velocity of light, it must have momentum similar to that of moving particle,

$\therefore$ we can write momentum of photon,

$$p = mc = \frac{h\nu}{c^2} \cdot c$$

$$\therefore \quad p = \frac{h\nu}{c} \quad \text{——} \quad (3)$$

But we know that

Velocity = frequency $\times$ wavelength.

$$c = \nu\lambda$$

$$\therefore \quad \nu = \frac{c}{\lambda} \quad \text{——} \quad (4)$$

from eq^{n}s (3) & (4) we have,

momentum of photon $= p = \frac{h\nu}{c} = \frac{h}{c} \cdot \frac{c}{\lambda}$

$$\therefore \quad \boxed{p = \frac{h}{\lambda} \quad \text{or} \quad \lambda = \frac{h}{p}} \quad \text{——} \quad (5)$$

From eq^n (5), we see that wavelength λ of the radiation is related to the momentum p of photon through Planck's Constant, h. Thus eq^n (5) Correlates wave characteristics λ with the particle characteristic p through the Planck's Constant h. λ is the wavelength Called De Broglie wavelength.

According to De Broglie idea, eq^n (5) must be true for photons as well as for the material particles. Thus if a particle of mass m is moving with a velocity v, it has a momentum $p = mv$ & the wavelength λ associated with the moving particle will be given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{—— (6)}$$

Thus according to De Broglie, a wavelength λ is associated with moving body of mass m & the wavelength of the wave is given by eq^n (6).

∴ (a moving body must shows wave characteristics. Such a wave associated with moving particle or with a body is called a matter wave or De Broglie wave.)

If the particle is moving with non-relativistic Velocity ($v << c$), then kinetic energy of the particle is

$$E = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Thus, $$E = \frac{p^2}{2m}$$

$$\therefore \quad p = \sqrt{2mE}$$

∴ eq^n (6) becomes,

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}} \quad \text{—— (7)}$$

– Illustrative Examples :-

1] Calculate the de Broglie wavelength of an $e^{\ominus}$ moving with a velocity 1/20 th of the velocity of light.

→ The de Broglie wavelength is given by,

$$\lambda = \frac{h}{mv}$$

Velocity (v) of $e^{\ominus}$ $= \frac{1}{20} \cdot C = \frac{1}{20} \times 3 \times 10^{8}$

$$v = 1.5 \times 10^{7} \text{ m/s.}$$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.5 \times 10^{7}}$$

$$= 0.4845 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 0.4845 \text{ A}^{\circ}}$$

2] Calculate de Broglie wavelength of an $e^{\ominus}$ which has kinetic energy equal to 15 eV.

→ we have,

$$\lambda = \frac{h}{\sqrt{2mE}}$$ where E = kinetic energy

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 15 \times 1.6 \times 10^{-19}}}$$

$$= 0.312 \times 10^{-9} \text{ m}$$

$$\boxed{\lambda = 3.12 \text{ A}^{\circ}}$$

3] Consider an $e^{\ominus}$ travelling with a velocity of 10^{7} cm/sec. Calculate de-Broglie wavelength of $e^{\ominus}$.

$(m = 9.11 \times 10^{-31}$ kg, $h = 6.625 \times 10^{-34}$ JS$)$.

→ The de-Broglie wavelength is given by,

$$\lambda = \frac{h}{p}$$

$$v = 10^{7} \text{ cm/sec} = 10^{7} \times 10^{-2} \text{ m/sec}$$

$$\therefore \quad \lambda = \frac{h}{mv}$$

$$= \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 10^{7} \times 10^{-2}}$$

$$= 0.7272 \times 10^{-34+31-5}$$

$$= 0.7272 \times 10^{-8} \text{ m}$$

$$= 72.72 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 72.72 \text{ A}^\circ}$$

4] Calculate the wavelength associated with a particle of mass 2 gm moving with velocity of 3 km/sec.

→ <u>Given</u> :- $m = 2 \text{ gm} = 2 \times 10^{-3}$ Kg

$v = 3 \text{ km/sec} = 3 \times 10^{3}$ m/sec.

The de-Broglie wavelength is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.625 \times 10^{-34}}{2 \times 10^{-3} \times 3 \times 10^{3}}$$

$$= \frac{6.625}{6} \times 10^{-34}$$

$$\boxed{\lambda = 1.104 \times 10^{-34} \text{ m}}$$

5] Calculate the de-Broglie wavelength of $e^{\ominus}$ moving with velocity of $1/10^{th}$ of velocity of light.

(mass of $e^{\ominus} = 9.1 \times 10^{-31}$ Kg).

→ <u>Given</u> :- $m = 9.1 \times 10^{-31}$ kg

$$v = \frac{1}{10} \times c = \frac{1}{10} \times 3 \times 10^{8} = 0.3 \times 10^{8} \text{ m/s}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.3 \times 10^{8}}$$

$$= \frac{6.625}{27.3} \times 10^{-34+31-8}$$

$$\boxed{\lambda = 0.242 \times 10^{-11}\ m}$$

6] Calculate the de-Broglie wavelength of free $e^{\ominus}$ whose energy is 100 eV.

→ Given :- $m = 9.1 \times 10^{-31}$ kg

$E = 100$ eV $= 100 \times 1.6 \times 10^{-19}$ J

The de-Broglie wavelength is given by,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2912 \times 10^{-50}}}$$

$$= \frac{6.625}{53.96} \times 10^{-34+25}$$

$$= 0.122 \times 10^{-9}\ m$$

$$\boxed{\lambda = 1.222\ A^{\circ}}$$

7] Find the de Broglie wavelength of neutron, whose energy is 1 eV? (Given: mass of neutron $= 1.676 \times 10^{-22}$ kg)

→ The de Broglie wavelength is given by,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.676 \times 10^{-22} \times 1 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{5.3632 \times 10^{-41}}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{0.53632 \times 10^{-40}}}$$

$$= \frac{6.625}{0.7323} \times 10^{-34+20}$$

$$\boxed{\lambda = 9.046 \times 10^{-14}\ m}$$

* Wave mechanics :-

- The Schrodinger wave equation :-

In quantum mechanics, we cannot talk of well-defined position & momentum simultaneously. We know that moving particles exhibit wave-like characteristics. A wave group is associated with a moving particle. A mathematical function which describes wave-group is the wave function $\psi(\vec{r},t)$. As particle (or a system of particles) moves under the action of external forces, the wave function changes with time. The motion of a particle is described by the wavefunction $\psi(\vec{r},t)$. The mathematical eq^n of motion in terms of $\psi(\vec{r},t)$. The eq^n is now called as Schrodinger's wave eq^n. The term 'wave mechanics' usually refers to the mechanics based on Schrodinger's wave eq^n.

The wave fu^n $\psi(\vec{r},t)$ associated with a particle is a function of space coordinates x, y, z & time 't'. If 'ψ' is to be associated with the particle, then it must be zero where the particle is not likely to be found. Similarly it must have a non-zero value where there is some possibility of finding the particle. Thus 'ψ' must be related to probability of finding the particle at a given point (x, y, z) & at given time (t).

However, 'ψ' itself Cannot be directly related to probability. Because, in general, wave fu^n 'ψ' which represents wave-group is obtained by Superposition of number of monochromatic waves. The addition of such monochromatic waves may be negative at a given Space point (x, y, z) but probability Can never be negative. ∴ Probability is always real & positive.

$$\therefore \quad |\psi|^2 = \psi \psi^*$$

is taken as measure of probability of finding the particle at the point (x, y, z) in Space at an instant 't'. The quantity $|\psi|^2$ is called the probability density. The probability that the particle be found Somewhere in a small volume element 'dV' around the given point at given instant is $|\psi|^2 \, dV$.

Larger the value of $|\psi|^2$, greater is the chance of finding the particle 'there at that instant'. As long as $|\psi|^2$ is not equal to zero, there is a definite chance of finding the particle at the given point at given time. The particle under consideration will always be found somewhere, total probability is always equal to unity.

$$\text{i.e.} \quad \int \psi \psi^* \, dV = 1.$$

The integral is carried out over the entire space. The above Condition on 'ψ' is called the normalization Condition.

– The Schrodinger wave Equation :-

In microscopic (nanoscopic) phenomena, we cannot talk of definite position & definite momentum for a particle at the same time. We have to talk in terms of probability. It is the wave function $\psi(x,y,z,t)$ which contains all the information about the probabilistic behaviour of the particle. Schrodinger formulated a wave eqn for $\psi(x,y,z,t)$. This eqn is called Schrodinger equation.

We know that a monochromatic wave propagating in the +ve x-dirn is represented by,

$$\psi(x,t) = A\exp[i(kx-\omega t)] \quad \text{—— ①}$$

where, A = amplitude of the wave.

This wave corresponds to a free particle with momentum $p = \frac{h}{\lambda}$ in the x-dirn. The variable 'k' & 'p' are related by $p = \hbar k$ where $\hbar = \frac{h}{2\pi}$.

Also $E = \hbar\omega$.

Energy E & momentum p are related by,

$$E = \frac{p^2}{2m}.$$

$\therefore$ eqn ① can be written as,

$$\psi(x,t) = Ae^{i(px - Et)/\hbar}$$

$$\therefore \psi(x,t) = Ae^{i\left(\frac{p}{\hbar}x - \frac{Et}{\hbar}\right)} \quad \text{—— ②}$$

Differentiating above eqn with respect to x we get,

$$\frac{\partial\psi}{\partial x} = A\frac{i}{\hbar}p\exp\left[i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)\right]$$

$$\therefore \frac{\partial\psi}{\partial x} = \frac{i}{\hbar}p\psi \quad \text{—— ③}$$

Differenting above eq^n ③ w.r.t. x we get,

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{ip}{\hbar}\left(\frac{\partial \psi}{\partial x}\right)$$

$$= \frac{ip}{\hbar}\left(\frac{ip\psi}{\hbar}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{i^2 p^2 \psi}{\hbar^2}$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2 \psi}{\hbar^2} \qquad \because i^2 = -1.$$

$$\therefore \boxed{p^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2}} \quad — (4)$$

Differentiating eq^n ② w.r.t. t, we get

$$\frac{\partial \psi}{\partial t} = -A \frac{i}{\hbar} E \exp\left[i \left(\frac{p}{\hbar} x - \frac{E}{\hbar} t \right) \right]$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi$$

$$\therefore E\psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$$

$$\therefore \boxed{E\psi = i\hbar \frac{\partial \psi}{\partial t}} \quad — (5) \quad \left(\because \frac{1}{i} = -i\right).$$

Total energy E of a particle is sum of K.E. i.e. $\frac{p^2}{2m}$ & potential energy V

$$\therefore E = K.E. + P.E.$$

$$\therefore E = \frac{p^2}{2m} + V$$

multiplying above eq^n by ψ,

$$E\psi = \frac{p^2\psi}{2m} + V\psi$$

from eq^{n}s (4) & (5), we get

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi}$$

This is called as Schrodinger's time dependent eq^n. It is one dimensional.

Two-dimensional Schrodinger time dependent eq^n is given by,

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) + V\psi}$$

Also, 3-dimensional eq^n is given by,

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V\psi}$$

- Schrodinger Time Independent Equation :-

Schrodinger time dependent eq^n is given by,

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V\psi$$

we will consider 1-dimensional Schrodinger wave eq^n.

$$\therefore \quad i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \quad \text{—— (1)}$$

This equation can be solved by using method of variable separation.

$$\therefore \quad \psi = \psi(x,t) = \psi(x)\,\phi(t) \quad \text{——— ②}$$

Substituting eq^n ② in eq^n ① we get,

$$i\hbar \frac{\partial}{\partial t}\left(\psi(x)\,\phi(t)\right) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\psi(x)\,\phi(t)\right) + V\psi(x)\,\phi(t)$$

$$\therefore \quad i\hbar\,\psi(x)\frac{\partial \phi(t)}{\partial t} = \phi(t)\left[\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V\psi(x)\right]$$

$$\therefore \quad \frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t} = \frac{-\hbar^2}{2m}\cdot\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + \frac{V\psi(x)}{\psi(x)}$$

$$\therefore \quad \frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t} = \frac{-\hbar^2}{2m\,\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V \quad \text{— ③}$$

In this eq^n, R.H.S. depends on x only & LHS depends only on t. However x & t are independent of each other. Hence, eq^n ③ can be true only if each side is equal to some constant. We denote this constant by E. It has the dimensions of energy of the system.

$$\therefore \quad E = \frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t}$$

$$\therefore \quad E = \frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V$$

$$\therefore \quad \boxed{\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\,\psi(x) = 0} \quad \text{— ④}$$

This eq^n is called as Schrodinger Time independent eq^n.

Eigenvalues & Eigenfunctions :-

Suppose "A" is an operator, & $f(x,y,z)$ is a function such that,

$$A\, f(x,y,z) = a\, f(x,y,z) \quad \text{——} \quad (1)$$

where 'a' is a scalar. In the above eq^n, operator 'A' has a typical action on 'f' which results in 'f' itself multiplied by a scalar. The eq^n of the above type is called an eigenvalue eq^n for operator A. The function $f(x,y,z)$ is called the eigenfunction ~~for~~ of A & 'a' is called the eigenvalue of A corresponding to eigenfunction f.

In quantum mechanics, we often deal with the Hamiltonian operator ($\hat{H}$). The eigenvalue eq^n for $\hat{H}$ is:

$$\hat{H}\,\psi(\hat{r}) = E\psi(\vec{r}).$$

This is nothing but time-independent Schrodinger equation. The eigenvalues & eigenfunctions are called energy eigenvalues & energy eigenfunctions. Similarly, for momentum operator we have momentum eigenvalues & momentum eigenfunctions. In fact, an operator is associated with every physical quantity & each such operator has its own set of eigenvalues & eigenfunctions.

Applications of Schrodinger Equation :-

1] Constant potential (Free Particle) (Electron in free Space) :-

Let us consider the motion of a particle in a field of constant potential V_0. Since force $F = -\nabla V$, constant potential means that there is no force acting on the particle. Such a free particle will keep on moving in a given dir^n (along X-axis or Y-axis or Z-axis). ~~wh~~ with constant momentum $p = mv$

& its total energy is :

$$E = \frac{1}{2} m v^2 + V_0.$$

The constant V_0 may be taken to be zero.
Then, $\therefore$ $E = \frac{1}{2} m v^2$.

classically the particle can move with any velocity between 0 to ∞. Hence, energy of the particle can have any value within this range. Thus, classically the energy spectrum of free particle is continuous.

Now, the quantum mechanical motion of the particle can be obtained by solving the Schrodinger equation. Since potential is time-independent $(V=0)$, we consider time-independent Schrodinger equation. Let us assume that the motion of the particle is along x-axis.

$$\therefore \left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\psi(x) \quad \text{—— (1)}$$

But $V=0$, in the present case.

$$\therefore \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x)$$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0 \quad \text{—— (2)}$$

Let $$K^2 = \frac{2mE}{\hbar^2} \quad \text{—— (3)}$$

Then, $$\frac{d^2\psi}{dx^2} + k^2\psi(x) = 0$$

Two linearly-independent solutions of this eq^n are e^{ikx} & e^{-ikx}

In case of steady states the time part of the wave function $\psi(x,t)$ is always given by,

$$e^{-iEt/\hbar}$$

Then, $\psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$

The energy of the free particle is,

$$E = \hbar\omega$$

Then,

$$\psi(x,t) = \psi(x) \cdot e^{-i\omega t}$$

∴ The two solutions of eq^n (2) are :-
$e^{i(kx-\omega t)}$ & $e^{-i(kx-\omega t)}$. The term $e^{i(kx-\omega t)}$ represents a monochromatic wave travelling in the +ve X-dir^n & the term $e^{-i(kx-\omega t)}$ represents a wave travelling in the negative X-dir^n.

Thus, for a particle moving in the forward X-dir^n, the wave associated with it is represented by,

$\psi(x,t) = Ae^{i(kx-\omega t)}$ where 'A' is a normalization constant. The wavelength associated with the particle is :

$$\lambda = \frac{2\pi}{K}$$

If we put $p = \hbar k$, then $\lambda = h/p$, as per the de Broglie's hypothesis,

Probability density is given by :-

$$\rho(x,t) = \psi^* \psi = A^* A = |A|^2 \text{ (constant)}.$$

Thus, The probability density is same everywhere & it is same at all times.

2] **Infinite Potential well :-**

Schrodinger time Independent differential eqⁿ is given by,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{--- (1)}$$

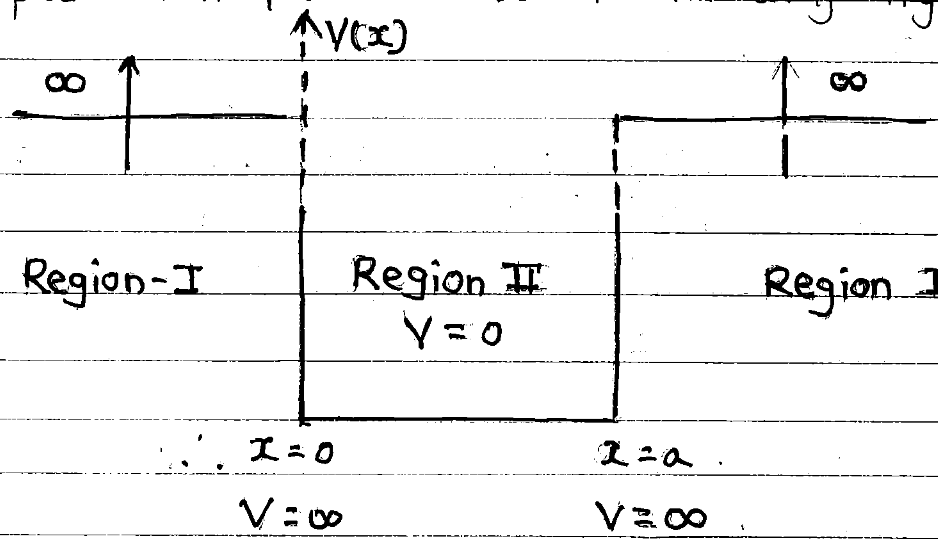
where,

ψ = wave function of a particle under motion.

E = Its kinetic energy & is moving under a force field whose potential is obtained by V.

This is also called Schrodinger Stationary state differential eqⁿ.

Let the particle is moving in a infinitely deeped well i.e. as shown in following fig.



Let us assume that a particle is trapped in a potential well with $V=0$ except that at the boundries $x=0$ & $x=a$, where it is infinitely large.

$\therefore \quad V = 0, \qquad 0 < x < a$

$\quad\;\; V = \infty, \qquad x = 0 \;\&\; x = a$

$\therefore$ Applying to eqⁿ (1),

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-0)\psi = 0.$$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \text{——} \quad (2)$$

Let $\frac{2mE}{\hbar^2} = k^2$

$\therefore$ eq^n (2) becomes,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

Let put $\frac{d}{dx} = D$

$$\therefore D^2\psi + k^2\psi = 0$$

$$\therefore (D^2+k^2)\psi = 0$$

since, ψ cannot be zero because it is the wave function of a particle.

$$\therefore D^2+k^2 = 0$$

$$\therefore D^2 = -k^2$$

$$\therefore D = \pm ik$$

$$\therefore \frac{d\psi}{dx} = ik\psi \quad \text{——} \quad (i)$$

$$\frac{d\psi}{dx} = -ik\psi \quad \text{——} \quad (ii)$$

By solving (i),

$$\frac{d\psi}{\psi} = ik\,dx$$

Integrating on Both sides,

$$\int \frac{d\psi}{\psi} = \int ik\,dx$$

$$\therefore \log_e \psi_1 = ikx + \log A$$

$$\therefore \log_e(\psi_1 - A) = ikx$$

$\therefore \log\left(\frac{\psi_1}{A}\right) = ikx$

$\therefore \psi_1 = Ae^{ikx}$

Similarly solving eq^n (ii) we get,

$\psi_2 = Be^{-ikx}$

$\therefore \psi = \psi_1 + \psi_2$

$\therefore \psi = Ae^{ikx} + Be^{-ikx}$

From the boundary condition,

$\psi \to 0$ as $x \to 0$

Because due to infinite potential well, particle does not cross well to L.H.S.

$\therefore$ Its wave function to the left of $x=0$, is $\psi = 0$ as $x = 0$.

$\therefore Ae^0 + Be^0 = 0$

$\therefore A + B = 0$ or $A = -B$

$\therefore \psi = Ae^{ikx} - Ae^{-ikx}$

$\therefore \psi = A(e^{ikx} - e^{-ikx})$

$\psi = A(e^{ikx} - e^{-ikx}) \times \frac{2i}{2i}$

$\therefore \psi = 2iA \sin kx$

$\therefore \boxed{\psi = A^* \sin kx}$ where, $A^* = 2iA$

Applying boundary condition (II) at $x=a$, since particle cannot crossing the region II because a particle cannot penetrate an infinite barrier.

$\therefore$ we must choose only the sine solution & define k such that $\sin kx$ goes to zero, at $x=a$.

$\therefore \quad A^* \sin ka = 0$

$\therefore \quad \sin ka = 0$

$\therefore \quad \boxed{ka = n\pi}$ with $n \neq 0$.

where $n = 1, 2, 3 \ldots$ referred to as quantum no

$\therefore \quad k^2 a^2 = n^2 \pi^2$

$\therefore \quad \frac{2mE}{\hbar^2} a^2 = n^2 \pi^2$

$$\therefore \quad \boxed{E = \frac{\hbar^2 \pi^2 n^2}{2m a^2}}$$

As $\hbar = \frac{h}{2\pi}$,

$$\therefore \quad E = \frac{n^2 \pi^2 \left(\frac{h^2}{4\pi^2}\right)}{2ma^2}$$

$$\therefore \quad \boxed{E = \frac{n^2 h^2}{8ma^2}}$$

$\therefore \quad E_n = \frac{n^2 \hbar^2 \pi^2}{2ma^2}$ with $n = 1, 2, 3 \ldots$

We have particle energies $E_1, E_2, E_3, E_4 \ldots$ are the allowed energy States.

$\therefore$ with energy E, it has a wave function ψ_1.

$\therefore \psi_1 = A^* \sin ka$ with $n = 1$, $ka = n\pi = \pi$

$\therefore \psi_1 = A^* \sin \pi$

$\psi_2 = A^* \sin 2\pi$ with $n = 2$, $ka = n\pi = 2\pi$

$\psi_3 = A^* \sin 3\pi$ with $n = 3$, $ka = n\pi = 3\pi$

& so on.

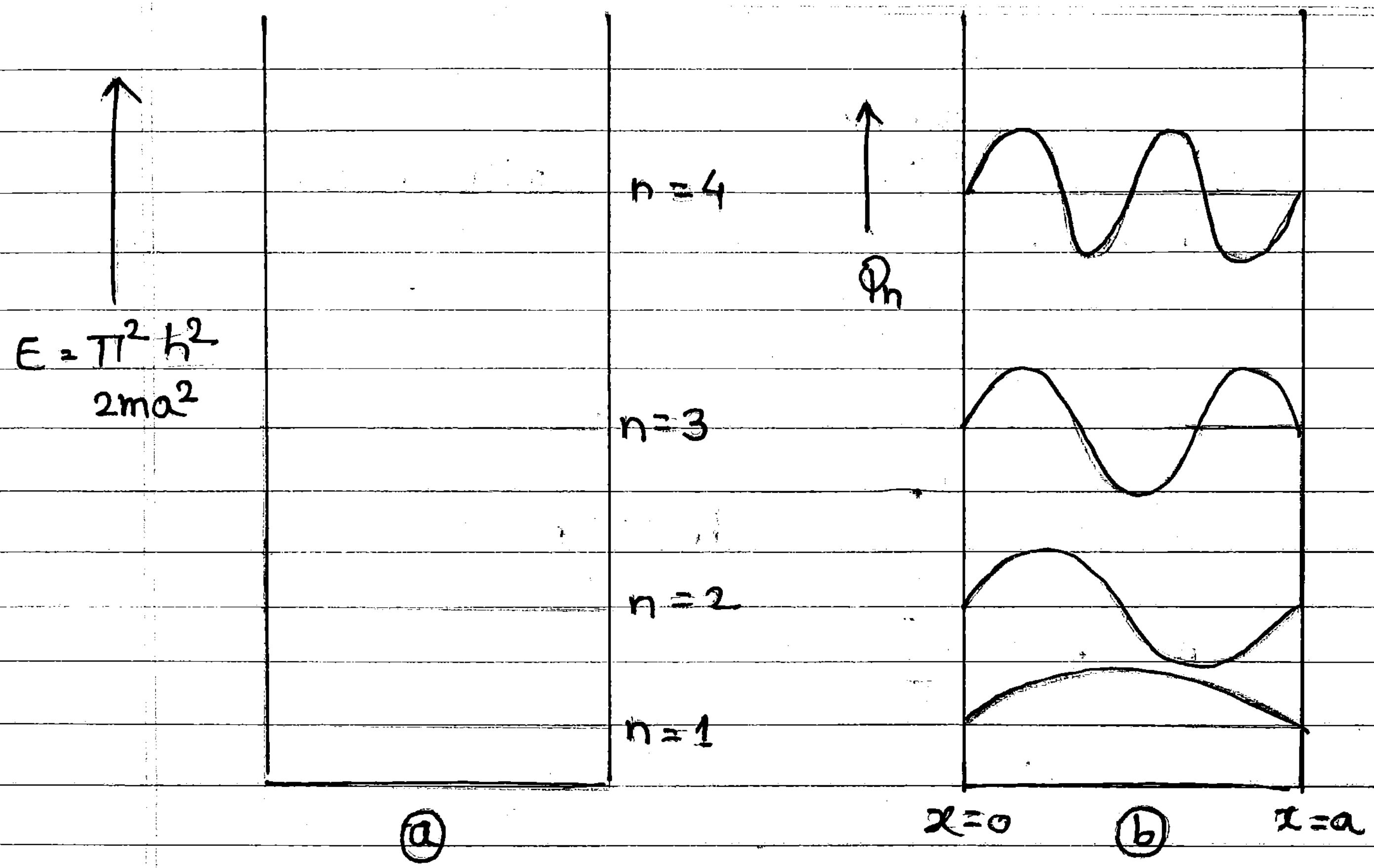


Fig :- Particle in an infinite Potential well.
(a) Four lowest discrete energy States. (levels).
(b) Corresponding wave functions.

3] <u>Qualitative Treatment of Square wave potential with special reference to tunneling Phenomenon:</u>
<u>(Quantum leak)</u>

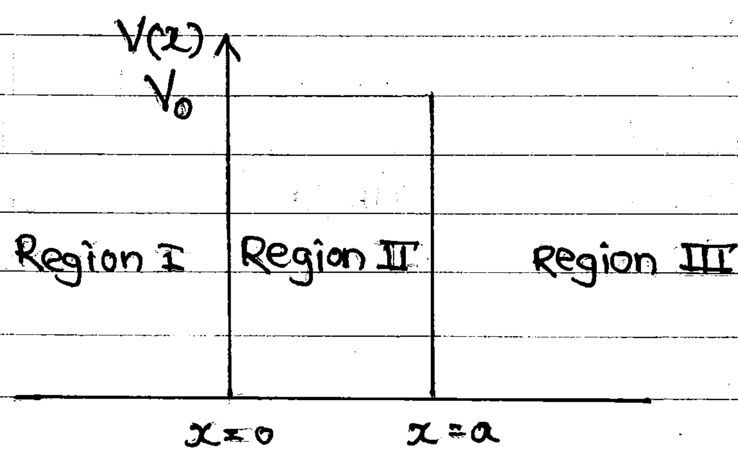


fig :- The potential barrier function

Consider the potential barrier function shown in the above fig. When the total energy of an incident particle is $E < V_0$, assume that we have a flux of incident particles originating on the negative x axis travelling in the $+x$ direction.

In the region I, Potential energy is zero.

$\therefore$ $e^{\ominus}$ will have all energy as kinetic energy, then $e^{\ominus}$ will force a barrier at $x=0$. The width of the barrier is a. on the other side of potential barrier, $x > a$, will have $P.E. = 0$.

Classically, $e^{\ominus}$ should bounce back from $x=0$ because total energy E of the $e^{\ominus}$ is $E < V_0$ but quantum mechanically, there is a possibility that $e^{\ominus}$ will tunnel through the potential barrier & appear on the other side.

To show this, we will solve the Schrodinger eq^n. To do this we divide the $e^{\ominus}$ motion into 3 regions Ist, IInd & IIIrd as indicated in fig.

We will solve Schrodinger eq^n in each region to obtain 3 wave functions in region I, II & III.

$\therefore$ Applying Schrodinger eq^n for Ist region,

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$$\therefore \frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad V = 0,\ x < 0$$

$$\therefore \boxed{\psi_I = A_1 \exp(jkx) + A_2 \exp(-jkx)}$$

where, $\dfrac{2mE}{\hbar^2} = k^2$

For region II,

$$\frac{d^2\psi_{II}}{dx^2} + \left(\frac{-2m}{\hbar^2}\right)\left(V_0 - E\right)\psi_{II} = 0$$

Let $\frac{2m}{\hbar^2}(V_0 - E) = \alpha^2$

$$\therefore \frac{d^2\psi_{II}}{dx^2} - \alpha^2\psi_{II} = 0$$

$$\therefore \boxed{\psi_{II} = B_1 \exp(j\alpha x) + B_2 \exp(-j\alpha x)}$$

For region III,

$$\boxed{\psi_{III} = C_1 \exp(jkx) + C_2 \exp(-jkx)}$$

with $k^2 = \frac{2mE}{\hbar^2}$ & $\alpha^2 = \frac{2m}{\hbar^2}(-E+V_0)$.

The region I contains $A_1 \exp(jkx)$ is an incident flux moving in the $+x$ direction. $A_2 \exp(-jkx)$ is moving in the $-x$ direction which is reflected wave.

In the region III, there is no reflected wave. $\therefore \boxed{C_2 = 0}$

Applying the boundary conditions for continuity of ψ at $x=0$ & $x=a$, we have two functions called as transmission coefficient & reflection coefficient.

We can calculate them as,

$$R = \frac{A_2^2}{A_1^2} = \frac{A_2 \cdot A_2^*}{A_1 \cdot A_1^*}$$

($*$ represents complex conjugate).

& Transmission Coefficient

$$T = \frac{C_1^2}{A_1^2} = \frac{C_1 \cdot C_1^*}{A_1 \cdot A_1^*}$$

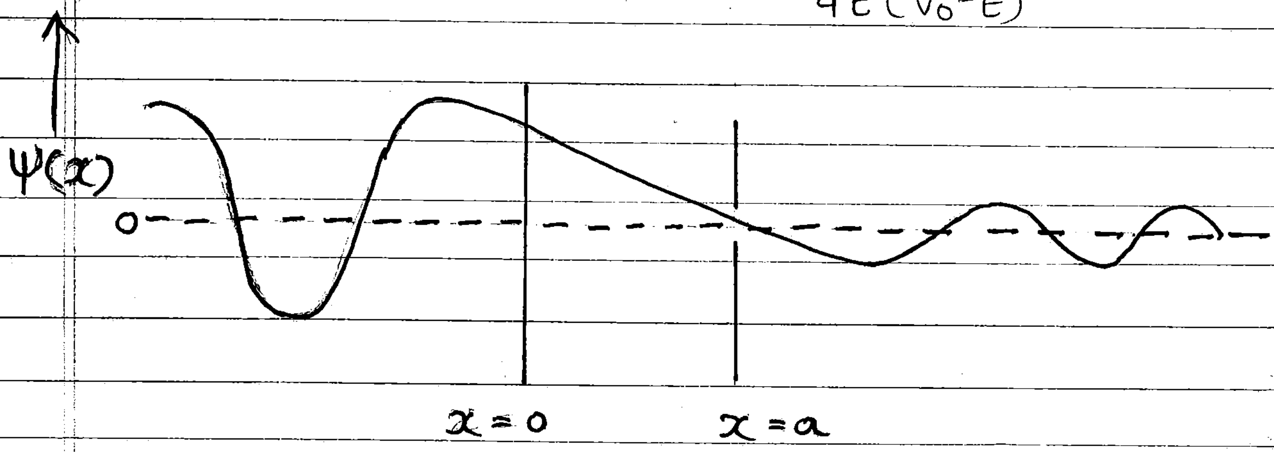
Then transmission coefficient is given by,

$$T = \frac{16E(V_0 - E)}{V_0^2} \exp(-2\alpha a).$$

or $\quad T = T_0 \exp(-2\alpha a).$

with $\quad T_0 = \frac{16E(V_0 - E)}{V_0^2}$

or $\quad T = \frac{1}{1 + \frac{V_0^2 \sinh^2 \alpha a}{4E(V_0 - E)}}$



<u>Fig:- wave functions through the potential barrier</u>

<u>Tunneling effect</u> :- When an $e^{\ominus}$ encounters a potential barrier of height V_0, greater than its energy E, there is finite probability that it will leak through barrier. This probability depends sensitively on the energy of $e^{\ominus}$ & width of a barrier. For wide potential barrier, the probability of tunneling is proportional to $\exp(-2\alpha a)$.

The wider the potential barrier, smaller the chance of $e^{\ominus}$ tunneling.

- Examples :-

1] Calculate the ground state energy of an $e^{\ominus}$ Confined to move freely between two ends Separated by 10 A°. ($m = 9.11 \times 10^{-31}$ kg).

→ Given: $m = 9.11 \times 10^{-31}$ kg

$a = 10 A° = 10 \times 10^{-10}$ m. $= 10^{-9}$ m.

The ground state energy of $e^{\ominus}$ is given by,

$$E_1 = \frac{\pi^2 \hbar^2}{2ma^2} \qquad \because \hbar = \frac{h}{2\pi}$$

$$\therefore \hbar^2 = \frac{h^2}{4\pi^2}$$

$$= \frac{\cancel{\pi^2} h^2}{8\cancel{\pi^2} ma^2}$$

$$= \frac{h^2}{8ma^2}$$

$$= \frac{(6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (10^{-9})^2}$$

$$= \frac{43.89 \times 10^{-68}}{72.88 \times 10^{-31-18}}$$

$$= 0.6022 \times 10^{-68+49}$$

$$\boxed{E_1 = 0.6022 \times 10^{-19} \text{ J}}$$

$$\therefore E_1 (eV) = \frac{0.6022 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ C}}$$

$$\boxed{E_1 = 0.37 \text{ eV}}$$

2] Calculate the ground state energy of a particle of mass 10 gm which is free to move between two ends separated by 10 A°.

→ Given : $m = 10\,gm = 10 \times 10^{-3}\,kg$

$\therefore m = 10^{-2}\,kg$

$a = 10\,A^{\circ} = 10^{-9}\,m.$

The ground state energy of a particle is given by,

$$E_1 = \frac{\pi^2 \hbar^2}{2ma^2} \qquad \because \hbar = \frac{h}{2\pi}$$

$$= \frac{\pi^2 h^2}{8\pi^2 ma^2} \qquad \therefore \hbar^2 = \frac{h^2}{4\pi^2}$$

$$= \frac{h^2}{8ma^2}$$

$$= \frac{(6.625 \times 10^{-34})^2}{8 \times 1 \times 10^{-2} \times (10^{-9})^2}$$

$$= \frac{43.89 \times 10^{-68}}{8 \times 10^{-2-18}}$$

$$= \frac{43.89}{8} \times 10^{-68+20}$$

$$\boxed{E_1 = 5.48 \times 10^{-48}\,J}$$

3] A marble ball of mass 50 gm is performing to & fro motion with steady velocity between two ends 1m apart, The period of motion is 10 seconds. What is the energy of the marble? Estimate the value of Corresponding quantum number (n).

→ When the marble Completes one cycle of motion the total distance covered is 2m.

$$\therefore \text{Velocity } v = \frac{\text{distance}}{\text{time}} = \frac{2}{10} = 0.2\,m/s.$$

$$\text{Energy } E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times (50 \times 10^{-3})(0.2)^2 = \frac{1}{2} \times 2 \times 10^{-3}$$

$$\boxed{E = 10^{-3}\,J}$$

Now, $E_n = \frac{\pi^2 n^2 \hbar^2}{2ma^2} = \frac{n^2 h^2}{8ma^2}$

$$\therefore n^2 = \frac{8ma^2 E_n}{h^2}$$

$$= \frac{8 \times 50 \times 10^{-3} \times (1)^2 \times 10^{-3}}{(6.625 \times 10^{-34})^2}$$

$$= \frac{400}{(6.625)^2} \times 10^{-6+68}$$

$$n^2 = 9.11 \times 10^{62}$$

$$\therefore \boxed{n \approx 3.01 \times 10^{31}}$$

* The Instruments had made possible bottom up approach possible in Semiconductor technology are:-

1] Scanning electron microscope (SEM)
2] Scanning tunneling microscope (STM).
3] Transmission electron microscope (TEM).
4) atomic force microscope (AFM).

* NEMS (Nanoelectromechanical systems):-

This approach employs both electrical & mechanical properties of nanostructures. The new generation of devices & systems based on this approach is Commonly referred to as nanoelectromechanical systems (NEMS). This electromechanical Concept may be used for the development of a new class of devices that includes nanomechanics, novel sensors, & a Variety of ~~a~~other ~~new~~ new devices functioning on the nanoscale.

* **Atoms & atomic orbital :-**

one - electron atom :-

Consider the behaviour of electron in Hydrogen atom. This $e^{\ominus}$ has to move with a attractive potential field called Coulombic potential energy,

$$V(r) = \frac{-e^2}{4\pi\epsilon_0 r}$$

where, e = magnitude of electronic charge &
ϵ_0 = permittivity of free space.

& force $(f) = \dfrac{-dV}{dr}$.

This potential function is spherically symmetric & leads to a 3-dimensional problem in spherical co-ordinates r, θ & ϕ.

$\therefore$ Schrodinger's wave eq^n to 3-dimensions will be,

$$\nabla^2 \psi(r,\theta,\phi) + \frac{2m}{\hbar^2}\left(E - V(r)\right)\psi(r,\phi,\phi) = 0 \quad \text{--- (1)}$$

The solution to eq^n (1) can be determined by the separation of variables technique.

$$\therefore \psi(r,\theta,\phi) = R(r)\,\phi(\theta,\phi)$$

where $\phi(\theta,\phi)$ is called angular probability density function, &

$R(r)$ is radial probability density function.

Applying all boundary conditions, the solution of eq^n (1) involves 3 quantum numbers called principal quantum number, orbital quantum number, & magnetic quantum number are denoted by n, l & m respectively.

The function $\phi(\theta, \phi)$ dependens on n, l, & m. whereas $R(r)$ depends on principle quantum no. n. The l values carry a special meaning. The first four l values are designated by 1st letter. They are termed as s (sharp), p (principal), d (diffuse) & f (fundamental).

The radial probability density is defined as the probability per unit radial distance. This behaviour implies that the probability of finding the $e^\ominus$ within a thin spherical shell close to the nucleus disappears for n = 1 & n = 0.

The maximum probability of finding the $e^\ominus$ at a particular distance from the nucleus is

$$r = a_0$$

which is called as Bohr radius.

∴ If the $e^\ominus$ is in 1s state, it spends most of the time at a distance a.

∴ $\psi_{n,l,m}(r, \theta, \phi)$ describes the wave function of $e^\ominus$. The possible energies of $e^\ominus$ with quantum no. n is given by,

$$E_n = \frac{-me^4}{8\epsilon_0^2 h^2 n^2} \qquad \text{OR}$$

$$E_n = \frac{-13.6\text{ eV}}{n^2} \qquad \text{with} \quad \frac{me^4}{8\epsilon_0^2 h^2} = 13.6\text{eV}$$

∴ we have, with n = 1, energy $E_1 = -13.6$ eV. Called as ground state, $E_2 = -3.40$ eV (n = 2) Called as 1st excitation state.

The $e^\ominus$ can only be excited to the next energy level if it is supplied by right amount of energy $E_2 - E_1$. The max^m probability for $n=2$ is at a distance $r = 4a_0$.

This particular Concept explain the absorption Spectra which is direct Consequence of quantization of energy. since the principal quantum no. determines the energy of $e^\ominus$ & also position of $e^\ominus$. Various Values of n Say 1, 2, 3 ... are termed as K, L, M, N as shells of $e^\ominus$s for a given n. l value gives to subshell. For e.g. 3S, 3p, 3d etc.

The maximum radial probability distribution is,

$\therefore r_{max} = n^2 a_0$ for $l = n-1$ with a_0 is Bohr radius.

Ch. II Ch. III :- Statistical Aspects for nanotechnology

* Gaussian Distribution :-

In probability theory & statistics, the normal distribution or Gaussian distribution is a continuous probability distribution that often gives a good description of data that cluster around the mean.

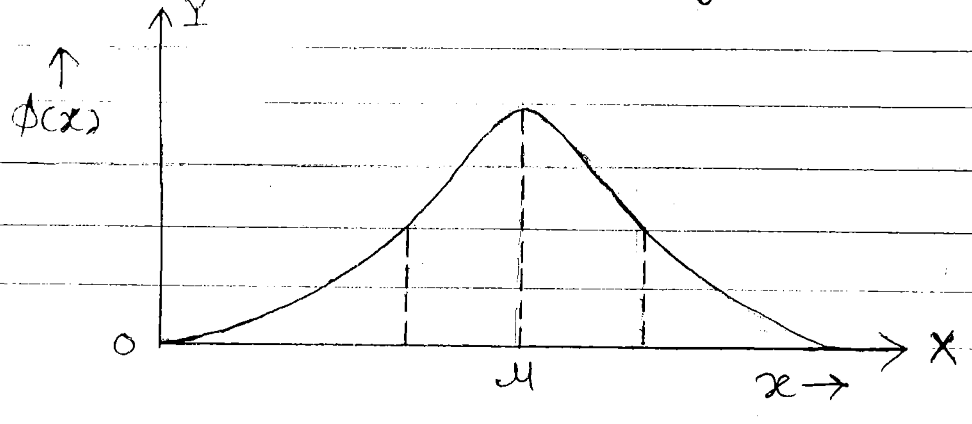
The graph of the associated probability density function is bell-shaped, with a peak at the mean, & is known as the Gaussian function or bell curve.

$$f(x) = \frac{1}{2\pi\sqrt{\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

where parameters μ & σ^2 are the mean & the variance. The distribution with $\mu = 0$ & $\sigma^2 = 1$ is called standard normal. The normal distribution is often used to describe, at least approximately & variable that tends to cluster around the mean.

- **Definition** : The simplest case of normal distribution is known as the standard normal distribution, described by the probability density function.

The constant in this expression ensures that the total area under the curve $\phi(x)$ is equal to one, & $\frac{1}{2}$ in the exponent makes the "width" of the curve. (measured as half of the distance betn the inflection points of the curve). & also equal to 1. In statistics, this function is denoted with the greek letter ϕ (phi),



For a standard normal distribution, $\mu = 0$ & $\sigma^2 = 1$.
The last part of the eq^n above shows that any other normal distribution can be regarded as a standard normal distribution that has been stretched horizontally by a factor of σ & then translated rightward by a distance μ. Thus, μ specifies the position of the bell's curve central peak & σ specifies the "width" of the bell curve.

The parameter μ is at the same time the mean, the median & the mode of the normal distribution. The parameter σ^2 is called the variance, as for any random variable, it describes how concentrated the distribution is around its mean. The square root of σ^2 is called the standard deviation & is the width of the density function.

- Properties :-

- Function $f(x)$ is symmetric around the point $x = \mu$, which is at the same time the mode, median & the mean of distribution. The inflection points of the curve occur one standard deviation away from the mean (i.e. at $x = \mu - \sigma$ & $x = \mu + \sigma$).

* Poisson Distribution :-

The Poisson distribution is a discrete probability distribution that expresses the probability distribution of events occuring in a fixed period of time if these events occur with a known average rate & independently of the time since the last event.

If the expected number of occurences in this interval is λ, then the probability that there are exactly K occurences (K being non-negative integer, $K = 0, 1, 2 \cdots$) is equal to

$$f(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where, e is the base of natural logarithm.
k is the no. of occurences of an event - the probability of which is given by the function.
k! is the factorial of k.
λ is a +ve real no., equal to the expected no. of occurences that occur during the given interval.

* Fermi - Dirac Statistics :-

Consider the interaction for which no two electrons can be in the same quantum state. i.e. the system must essentially satisfy Pauli's Exclusion Principle.

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{KT}\right)} \quad \text{—— (I)}$$

where, E_F is called the Fermi Energy. The f(E) is called Fermi - Dirac distribution or probability function & gives the probability that a quantum state at energy E will be occupied by an electron.

Let $T = 0^\circ K$ & consider the case when $E < E_F$.
The exponential term in the above eqn becomes,

$$\exp(E-E_F/KT) \Rightarrow \exp(-\infty) = 0$$

The resulting distribution function f(E) is,

$$f(E<E_F) = \frac{1}{1+0} = 1.$$

Again let $T = 0^\circ k$ & consider the case when $E > E_F$.
The exponential term in the distribution function becomes,

$$\exp(E-E_F/KT) \Rightarrow \exp(+\infty) = +\infty$$

The resulting distribution function f(E) is,

$$f(E>E_F) = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

The Fermi-Dirac distribution function for $T = 0K$ is plotted in the following fig :-

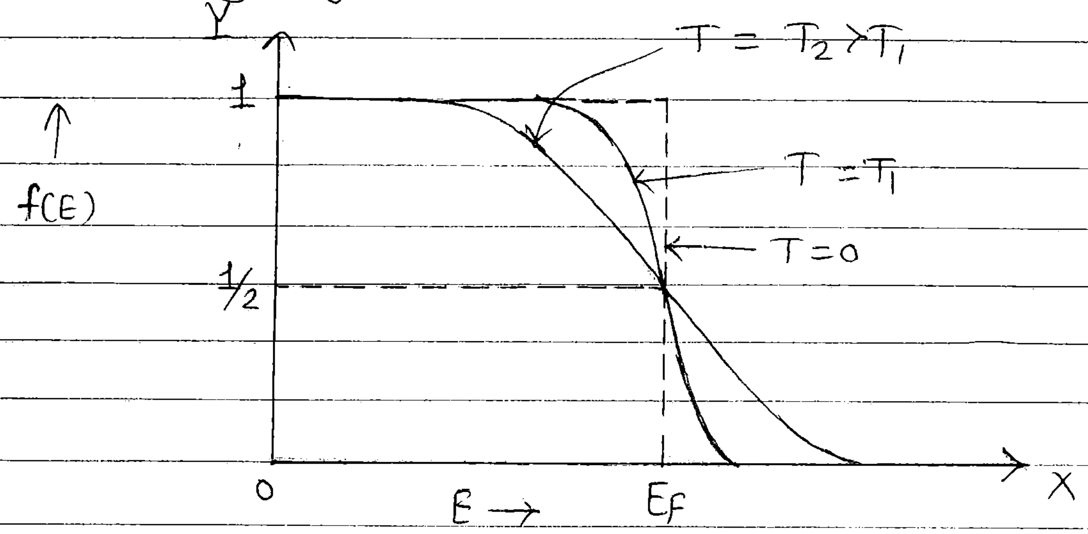


Fig:- The Fermi probability fuⁿ $f(E)$ v/s energy for different temperatures (E).

This plot shows that, for $T = 0K$, the $e^{\ominus}s$ are in their lowest possible energy states. The probability of a quantum state being occupied is unity for $E < E_F$ & the probability of a state being occupied is zero $E > E_F$. All $e^{\ominus}s$ have energies below the fermi energy at $T = 0K$.

Consider a certain amount of thermal energy (temp.) increases above $T = 0K$. $e^{\ominus}s$ gain some amount of therma energy so that some $e^{\ominus}s$ can jump to higher energy levels, the distribution of $e^{\ominus}s$ among the available energy states will change.

The change in the $e^{\ominus}$ distribution among energy levels for $T > 0K$ can be seen by plotting the Fermi-Dirac distribution function.

If we let $E = E_F$ & $T > 0K$,
the eqⁿ (I) becomes,

$$f(E = E_F) = \frac{1}{1 + \exp(0)} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of state being occupied at $E = E_F$ is $\frac{1}{2}$.

For temperature above absolute zero, there is non-zero probability that some energy states above E_F will be occupied by electrons & some energy states below E_F will be empty.

* Bose-Einstein Statistics :-

The Bose-Einstein distribution applies when the particles are distinguishable, but can occupy the same available states i.e. it applies to quantum particles that do not obey the exclusion principle.

e.g. Photons & phonons.

The Bose-Einstein distribution is,

$$f(E) = f(E, \mu, T) = \frac{1}{e^{\frac{E-\mu}{KT}} - 1} \quad \text{--- (1)}$$

At the large values of energy, all the above Fermi-Dirac distribution & Bose-Einstein distribution are equal. This is because of the de-Broglie wavelength. As energy increases, the de-Broglie wavelength decreases & for avery high energies, the de-Broglie wavelength is vanishingly small, such that the particle becomes like a classical particle.

Using Fermi-Dirac distribution, the probability of a level being occupied by an $e^{\ominus}$ is $f(E)$, then the probability of a level not being occupied an $e^{\ominus}$ (i.e. being occupied by a hole) is,

$$1 - f(E) = \frac{1}{e^{\frac{E_F - E}{KT}} + 1} \quad \text{--- (2)}$$

since, $$\frac{1}{e^{\frac{E-E_F}{KT}} + 1} + \frac{1}{e^{\frac{E_F - E}{KT}} + 1} = 1$$

* Maxwell - Boltzmann Statistics :-

In Statistical mechanics, Maxwell - Boltzmann statistics describes the statistical distribution of material particles over various energy states in thermal equillibrium. When the temperature is high enough & density is low enough to render quantum effects negligible.

The expected number of particles with energy ϵ_i for Maxwell - Boltzmann Statistics is N_i where,

$$N_i = N \frac{g_i}{e^{(\epsilon_i - \mu)/kT}} = N \frac{g_i e^{-\epsilon_i/kT}}{Z}$$

where, N_i = no. of particles in state i

ϵ_i = energy of the i-th state.

g_i = the degeneracy of energy level i, the no. of particles states (excluding the "free particle" state), with energy ϵ_i

μ = chemical potential.

k = Boltzmann's constant.

T = absolute temp.

N = total no. of particles $= N = \sum_i N_i$

Z = Partition function $= \sum_i g_i e^{-\epsilon_i/kT}$

Equivalently, the distribution is sometimes expressed as,

$$\frac{N_i}{N} = \frac{1}{\exp\left(\frac{\epsilon_i - \mu}{kT}\right)} = \frac{e^{-\epsilon_i/kT}}{Z}$$

where, index i refers a particular state rather than set of all states with energy ϵ_i.

Fermi - Dirac & Bose - Einstein statistics apply when quantum effects are important & the particles are "indistinguishable". Quantum effects appear if the Concentration of particles $(N/V) \geq n_q$. Where N_q is the quantum Concentration, ~~for which n_q~~ the interparticle distance is equal to the thermal de Broglie wavelength So that the wavefunctions of the particles are touching but not overlapping. Fermi-Dirac statistics applies to fermions (particles that obey Pauli Exclusion principle) & Bose-Einstein statistics apply to bosons. As the quantum concentration depends on temperature, most systems at high temperature obey the classical (Maxwell-Boltzmann) limit unless they have a very high density.

Both Fermi-Dirac & Bose-Einstein become Maxwell-Boltzmann statistics at high temperature or at low ~~temperature~~. Concentration.

* Time & length Scales of the $e^{\ominus}$s in Solids :-

I] Length Scales :-

1] $e^{\ominus}$ fundamental length in Solids :-

Consider an $e^{\ominus}$ as almost free particle, by assigning to the $e^{\ominus}$ an effective mass that may differ from the mass of $e^{\ominus}$ in Vacuum.

The fundamentally important length is de Broglie wavelength of $e^{\ominus}$ in a solid. For a free particle this length is given by,

$$\lambda = \frac{2\pi\hbar}{p}.$$

For an $e^{\ominus}$ in a semiconductor nanostructure with the effective mass m^*, the de Broglie wavelength λ is typically greater than that of free $e^{\ominus}$ λ_0.

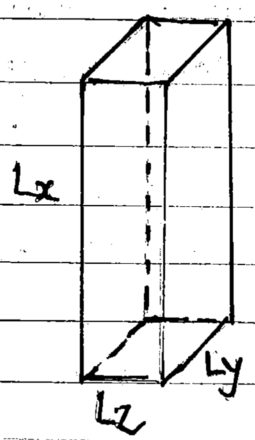
$$\therefore \lambda = \frac{2\pi\hbar}{p} = \frac{2\pi\hbar}{\sqrt{2m^*E}} = \lambda_0 \sqrt{\frac{m_0}{m^*}}$$

where, $\lambda_0 = \dfrac{2\pi\hbar}{\sqrt{2m_0E}}$.

where $E = e^{\ominus}$ energy & m_0 = mass of the $e^{\ominus}$ in Vacuum

2] Size of a device & $e^{\ominus}$ spectrum quantization :-

Let a geometrical size of a semiconductor sample, $L_x \times L_y \times L_z$ & assume that $L_z < L_y < L_x$



Since only an integer number of half wave of the $e^{\ominus}$s can fit into any finite system, instead of continuous energy spectrum, & a continuous number of the $e^{\ominus}$s states, we obtain a set of discrete $e^{\ominus}$ states & energy levels, each of which is characterized by the corresponding number of half-wavelengths. This is referred to as quantization of $e^\ominus$ motion.

$e^\ominus$s in Solid-State devices are Subjected to Scattering by Crystal imperfections, impurities, lattice vibrations, interface roughness, etc. These scattering processes are divided into two groups :-

elastic & inelastic.

In classical physics, an elastic collision leads to a change only in the particle momentum. & does not destroy the phase of $e^\ominus$. (After an elastic collision, the energy remains unchanged & the $e^\ominus$ wavefunction $\Psi(r,t)$ Consists of different Components.)

In an inelastic collision, both the momentum & the energy change. Inelastic scattering produces $e^\ominus$ waves with different energies & the resulting wavefunction has a Complex dependence on both position & time. the beating of different wave Components in time washes out the Coherence effects.

3] Classification of transport regime :-

— Quantum Regime $\longrightarrow$ Intercontact distance, L_x is Comparable to the $e^\ominus$ wavelength, $L_x \leq \lambda$.

— Mesoscopic regime $\longrightarrow$ Intercontact ~~length~~ distance is less than the dephasing length, $L_x < l_\phi$.

— classical regime (one, two & 3-dimensional $e^\ominus$ transport) $\longrightarrow$ Intercontact distance exceeds the dephasing length, $L_x > l_\phi$

classical ballistic regime, $l_e \geq L_x$

Quasiballistic regime (energy - Conserving):
$L_E \geqslant L_x \geqslant l_e, l_\phi$

Transverse size effects :
effect related to the mean free path,
$L_z, L_y \sim l_e$
diffusion effects, $L_z, L_y \sim L_E$

II] Time Scales :-

There are two fundamental times defining the character of $e^{\ominus}$ - transport behavior :-

i] the time between two successive Scattering events, or the Scattering time, τ_e; &

ii] the time which characterizes the duration of Scattering event τ_s.

Under ordinary conditions, $\tau_e \gg \tau_s$.
It is usually assumed that the scattering event is instantaneous i.e., $\tau_s \to 0$.

In classical regimes, the characteristic times & their relationships to the device sizes determine temporal & frequency regimes of device operation.

e.g. the transit time $t_{tr} = L_x / v$ determines the duration of signal propagation through a device. v is the $e^{\ominus}$ velocity.
$\therefore$ t_{tr} defines speed limit of device. The device cannot effectively operate in the time range less than t_{tr}.

In quantum mechanics, if external potentials are time-independent, the $e^{\ominus}$s are in stationary states.
The temporal evolution of a stationary state is always determined by an exponential factor $\exp\left[-i(E/\hbar)t\right]$.

If the alternating external field of an angular frequency ω is applied to the stationary $e^{\ominus}$ system the response of the $e^{\ominus}$ system may be referred to one of the following 3 different regimes depending on the freq^n of the external field.

1] Ultra-high (quantum) frequencies :-

If $\hbar\omega$ is comparable to the stationary $e^{\ominus}$ energy state E, the nature of the $e^{\ominus}$'s response will be quantum-mechanical. Only transitions between states with the energy difference $\Delta E = \hbar\omega$ are allowed. If E is quantized, the interaction is possible only at resonance frequencies. By varying device size, one can vary the energy spectrum & as a result, change the frequency properties over a wide range.

If $\hbar\omega << E$, the $e^{\ominus}$'s response to an alternating field is classical. In the classical picture, the external alternating field will cause periodic $e^{\ominus}$ acceleration & deceleration. Scattering interrupts these acceler^{n}s & decelerations

2] High (classical) frequencies :-

If $\omega\tau_e >> 1$, the $e^{\ominus}$ motion during one period is not interrupted by scattering. In accordance with classical mechanics, the $e^{\ominus}$ momentum oscillates with a phase opposite to that of field.

3] Low frequencies :-

If $\omega\tau_e << 1$, the $e^{\ominus}$ undergoes many scattering events during one period of the external field. Multiple scattering during the period brings the $e^{\ominus}$ into quasi-stationary state which follows the oscillations of the external field. The $e^{\ominus}$ momentum oscillates in phase with the field.

* Density of States of electrons :-

The density of states of a System describes the number of states at each energy level that are available to be occupied. A high density of states at a specific energy level means that there are many states available for occupation. A density of states of zero means that no states can be occupied at that energy level.

In quantum mechanical systems, not all waves, or wave-like particles, are allowed to exist. In Some Systems, the interatomic spacing & the atomic charge of the material allows only $e^{\ominus}$s of certain wavelengths to exist. In other Systems, the Crystalline Structure of the material allows waves to propagate in one direction, while suppressing wave propagation in another direction. Waves in quantum mechanical System have Specific wavelengths & can propagate in specific directions & each waves occupies a different mode or state. Because many of these states have the same wavelength & therefore share the same energy, there may be many states available at certain energy levels.

e.g. The density of states of $e^{\ominus}$s in semiconductor :-

For $e^{\ominus}$s at the Conduction band edge, Very few States are available for the $e^{\ominus}$ to occupy. As the $e^{\ominus}$ increases in energy, the $e^{\ominus}$ density of states increases & more states become available for occupation. However, because there are no states, available for $e^{\ominus}$s to occupy within the bandgap, $e^{\ominus}$s at the Conduction band edge must lose at least E_g of energy in order to transit to another available State.

* Electron transport in Metals :-

In metals, the bands either overlap or are only partially filled. Thus $e^{\ominus}$s & empty energy States are Intermixed within the bands so that $e^{\ominus}$s can move freely under the influence of an electric field. metals have a high electric Conductivity.

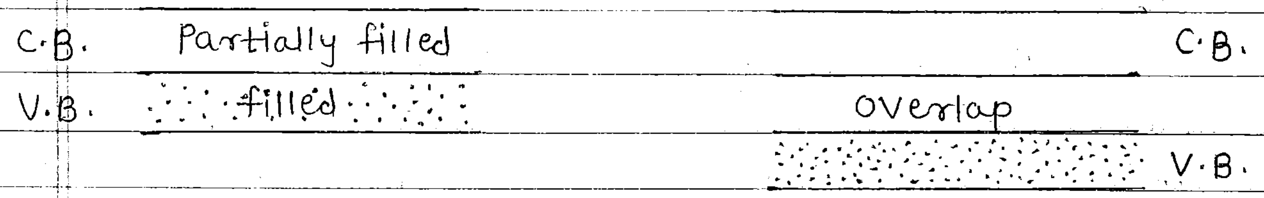


fig:- Typical band structure at 0°K.

* Electron transport in Semiconductors :-

We can observe that, for $e^{\ominus}$s to experience acceleration in an applied electric field, they must be able to move into new energy states. This implies there must be energy states (allowed energy States which are not occupied by $e^{\ominus}$s) available to the $e^{\ominus}$s.

The S_i band Structure is such that the Valence band is Completely filled with $e^{\ominus}$s at 0°K & the Conduction band is empty. Then there can be no charge transport within the Valence band, since no empty States are available into which $e^{\ominus}$s can move. Also there are no $e^{\ominus}$s in the Conduction band, so no charge transport can take place there either. The S_i has high resistivity typical of insulators.

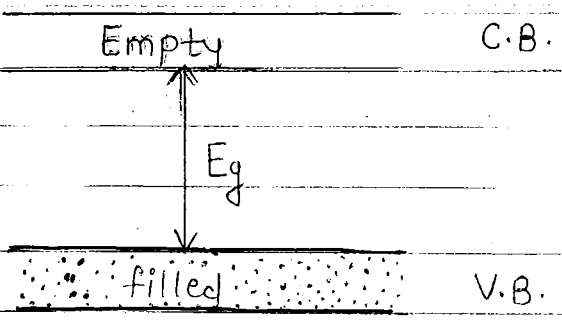
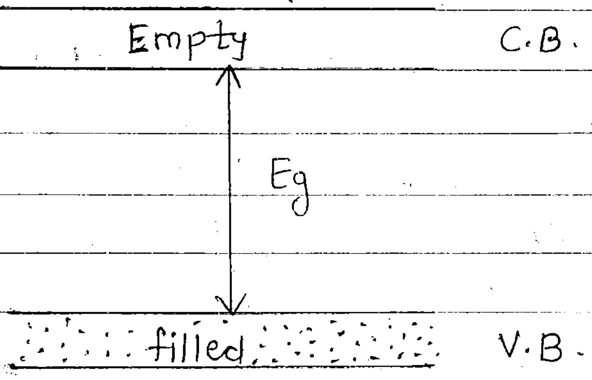


fig:- Typical band structure at 0°k.

Semiconductors at 0°k have basically the same structure as insulators. A filled V.B. separated from an empty C.B. containing no allowed energy states. The difference lies in the size of band gap E_g, which is much smaller in semiconductors than in insulators.

e.g. The semiconductor Si has a band gap of about 1.1 eV compared with 5 eV for diamond. The relatively small band gaps of semiconductors allow for excitation of $e^{\ominus}$s from the lower band (V.B.) to upper band (C.B.) by reasonably amount of thermal or optical energy.

At room temp., a semiconductor with 1eV band gap will have significant no. of $e^{\ominus}$s excited thermally across the energy gap into the CB whereas an insulator with band gap = 10 eV will have negligible no. of such excitations. Thus an important difference betn semiconductors & insulators is that the no. of $e^{\ominus}$s available for conduction can be increased greatly in semiconductors by thermal or optical energy.



Insulators

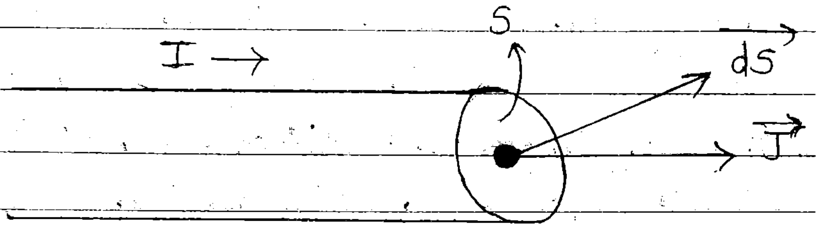
~~Ch. III~~ Ch. I : Essential Electromagnetics for Nanotechnology

* **Lorentz force – motion equation of charged particle in EM fields :-**

The current is the rate of flow of charge through the cross-section of a conductor.
An electric current in a conductor is due to moving charges, which move with some drift velocity. Generally, we consider the currents flowing in thin wires, but they may flow in large sheets or conductors of large volume. Thus ~~total~~ current is given by the rate of passage of charge across a specified area.

But at different points of big conductors of irregular shapes 'Local currents' may not be the same. In such situations, the concept of current density $\vec{J}$ is used.

$\vec{J}$ is the quantity of charge passing through per unit area & per unit time through an element of surface $\vec{ds}$ at right angles to the flow.



The current density $\vec{J}(\vec{r})$ at any point is in the direction in which a +ve charge moves at that point. $\vec{J}$ is assumed to be due to the motion of +ve charges. But if –ve charges are moving then $\vec{J}$ will be in the direction opposite to the actual charge motion.

The current I is the flux of $\vec{J}$ over the surface $\vec{S}$, the cross-sectional area of the conductor
i.e.

$$I = \int_S \vec{J} \cdot \vec{dS} \qquad \text{——— (1)}$$

If the surface over which the integration is carried out, is the normal cross-section of the conductor & $\vec{J}$ is constant over the cross-section then,

$$I = \int_S \vec{J} \cdot \vec{ds}$$

$$= J \int_S \vec{ds}$$

$$\therefore \quad I = JS$$

$$\vec{I}\, d\vec{I} = JS\, d\vec{I}$$

$$= \vec{J}\, dV$$

So that,

$$\vec{df} = (\vec{J} \times \vec{B})\, dV$$

$$\therefore \quad \frac{\vec{df}}{dV} = \vec{J} \times \vec{B} \quad \text{———} \quad (2)$$

But if there are n charged particles per unit volume, each having a charge q & moving with velocity $\vec{V}$ then,

$$\vec{J} = n q \vec{V}$$

Substituting in eq^n (2),

$$\frac{\vec{df}}{dV} = n q \vec{V} \times \vec{B}$$

or

$$\frac{1}{n} \frac{\vec{df}}{dV} = q (\vec{V} \times \vec{B})$$

since $n dV$ represents the total number of charged particles in volume dV, then term $\frac{1}{n}\frac{df}{dV}$ represents the force on an indivisual charge. Denoting it by $\vec{f}$ we have,

$$\boxed{\vec{F} = q (\vec{V} \times \vec{B})} \quad \text{———} \quad (3)$$

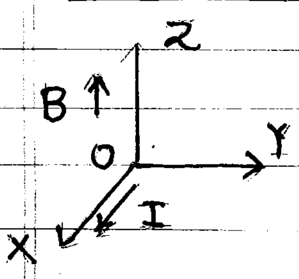
This is known as Lorentz force & is perpendicular to both the velocity $\vec{V}$ & magnetic induction $\vec{B}$.

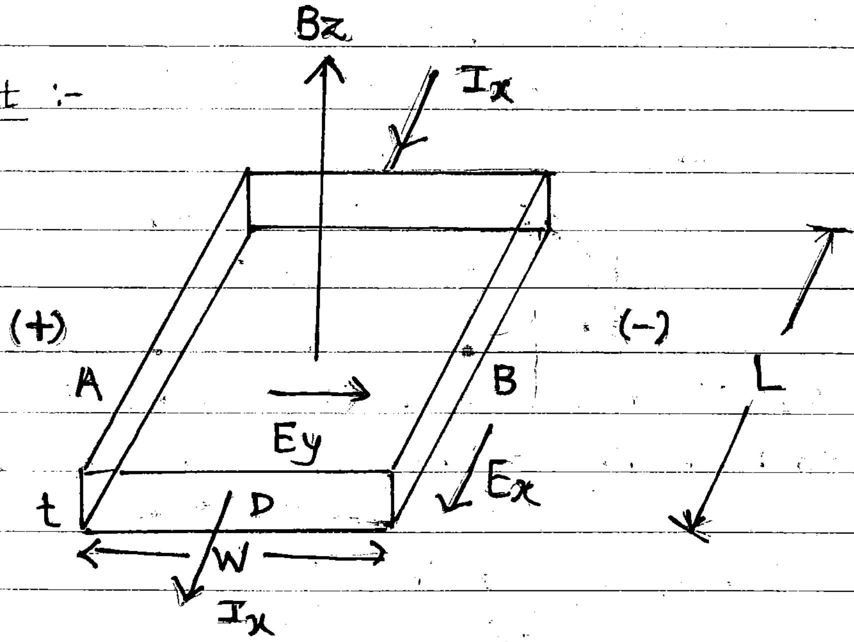
However if the charged particle is moving in both electric & magnetic fields, in addition to the above magnetic force, it will also experience an electric force $q\vec{E}$. In such cases, the total force acting on the charged particle will be,

$$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$$

$$\boxed{\vec{F} = q[\vec{E} + (\vec{V} \times \vec{B})]} \quad \text{——— (4)}$$

* Hall Effect :-





Hall effect :- If a magnetic field is applied perpendicular to the direction of current, a voltage is developed across the conductor at right angles to the both the direction of current flow & that of magnetic field. This voltage is known as Hall voltage & phenomenon is called as Hall effect.

If a magnetic field is applied perpendicular the direction in which holes drift in a P-type bar, The path of the holes tends to be deflected towards the bottom surface of the specimen & will accumulate there.

Using vector notation, the total force on a single hole due to the electric & magnetic field is

$$F_x = q(E_x + V_x B_z) \quad \text{—— (1)}$$

In the y-dir^n the force is,

$$F_y = q(E_y - V_x B_z) \quad \text{—— (2)}$$

From eq^n (2), unless an electric field E_y is established along the width of the bar, each hole will experience a net force in the y-dir^n, due to the qV_xB_z product. ∴ To maintain a steady state flow of holes down the length of the bar, the electric field E_y must just balance the product $V_x B_z$.

$$E_y = V_x B_z \quad \text{—— (3)}$$

So that the net force F_y is zero. Physically this electric field is set up when the magnetic field shifts the hole distribution slightly in the y-dir^n. once the electric field E_y becomes as large as V_xB_z, no net lateral force is experienced by the holes as they drift along the bar. The establishment of the electric field E_y is known as Hall effect. & the resulting Voltage

$V_{AB} = E_y W$ is called the Hall voltage.

If we use the expression derived in eq^n (2) for the drift velocity (using $+q$ & P_o for holes), the field E_y becomes,

$$J_x = q P_o V_x$$

∴ from eq^n (3),

$$E_y = V_x B_z$$

$$\therefore \quad E_y = \frac{J_x}{q P_o} B_z$$

$$E_y = R_H J_x B_z$$

where, $$R_H = \frac{1}{q P_o} \quad \text{—— (4)}$$

Thus the Hall effect is proportional to the product of the current density & the magnetic flux density. The proportionality constant

$$R_H = \frac{1}{qP_0}$$ is called Hall Coefficient.

A measurement of the Hall voltage for a known current & magnetic field yields a value of the hole concentration P_0,

$$P_0 = \frac{1}{qR_H}$$

$$= \frac{J_x B_z}{q E_y}$$

$$= \frac{(I_x/Wt) B_z}{q(V_{AB}/W)}$$

$$E_y = \frac{I_x B_z}{q t V_{AB}} \quad \text{——— (5)}$$

$$\boxed{V_{AB} = \frac{I_x B_z}{q t E_y}}$$

since all the quantities in the RHS of eq^n (5) can be measured. The Hall effect can be used to give quite accurate value for carrier concentration.

- Applications of Hall effect :-

1] To calculate the carrier concentration in a given conductor. (n_0, P_0).

2] To find out the type of semiconductor (p-type or n-type).

* Maxwell's Equations :-

1] Maxwell's Equations in Differential Form :-

In 1873, Prof. Jems Maxwell, England assembles the Ampere, Faraday, & Gauss's laws into a set of 4 equations Called Maxwell's equations.

Maxwell eq^{n}s are the 4 fundamental eq^{n}s of electromagnetism & are differential forms of the laws of electricity & magnetism.

a] Gauss's law in electrostatics :-

For the electric field of charge yields,

$$\overline{\nabla} \cdot \overline{D} = \rho$$

where, $\overline{D}$ = electric displacement in C/m^2.

ρ = free charge density in C/m^3.

b] Gauss's law for magnetic field yields,

$$\overline{\nabla} \cdot \overline{B} = 0$$

where $\overline{B}$ = magnetic induction in wb/m^2.

This is Gauss's law of magnetic induction.

c] Ampere's law in circuital form for the magnetic field accompanying a current when modified by maxwell yields,

$$\overline{\nabla} \times \overline{H} = \overline{J} + \frac{\partial D}{\partial t}$$

where, $\overline{H}$ = magnetic field intensity in A/m

& $\overline{J}$ = current density in A/m^2.

d] Faraday's law in circuital form for the electromotive force produced by the rate of change of magnetic flux linked with the path yields,

$$\overline{\nabla} \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

where, $\overline{E}$ = electric field intensity in V/m.

Thus, Maxwell's 4 fundamental eqns in differential form are —

$\overline{\nabla} \cdot \overline{D} = \rho$ — Gauss's law in electrostatics.

$\overline{\nabla} \cdot \overline{B} = 0$ — Gauss's law in magnetostatics.

$\overline{\nabla} \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$ — Differential form of Faraday's law.

$\overline{\nabla} \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$ — Modified Ampere's law.

2] Maxwell's eqns in Integral Form :-

a] We know that the 1st Maxwell's eqn in differential form is,

$$\overline{\nabla} \cdot \overline{D} = \rho$$

Integrating this eqn over a volume V we get,

$$\int_V \overline{\nabla} \cdot \overline{D} \, dv = \int_V \rho \, dv \quad \text{— } (1)$$

But $\int_V \rho \, dv = Q$

Applying Gauss's divergence theorem to the L.H.S of eqn (1) to convert volume integral into the surface integral.

$$\int_S \overline{D} \cdot d\overline{S} = \int_V \overline{\nabla} \cdot \overline{D} \, dv$$

$\therefore$ eqn (1) becomes,

$$\int_S \overline{D} \cdot d\overline{S} = Q \quad \text{— } (2)$$

where, Q = net charge contained within volume V.

& S = Surface bounding volume V.

"The outward flux of the electric displacement $\overline{D}$ through any closed surface 'S' is equal to the net charge within the volume."

This is Gauss's law in electrostatics.

b] The 2^{nd} maxwell's eq^n in differential form is,

$$\overline{\nabla} \cdot \overline{B} = 0$$

Integrating this eq^n over a volume V we get,

$$\int_V \overline{\nabla} \cdot \overline{B} \, dV = 0$$

Using Gauss's divergence thm we have,

$$\int_V \overline{\nabla} \cdot \overline{B} \, dV = \int_S \overline{B} \cdot d\overline{S}$$

$$\therefore \int_S \overline{B} \cdot d\overline{S} = 0 \quad \text{—— (3)}$$

Thus, the outward flux of magnetic induction $\overrightarrow{B}$ through any closed Surface S is equal to zero.

The physical meaning of the statement $\overline{\nabla} \cdot \overline{B} = 0$ is that the magnetic field lines do not diverge out from a point. Instead they close back on themselves. In other words, "there are no free sources of $\overrightarrow{B}$." The statement div. $\overrightarrow{B} = 0$ ($\overline{\nabla} \cdot \overline{B} = 0$) thus directly leads to the conclusion that the magnetic monopoles do not exist.

c] The 3^{rd} Maxwell's eq^n in differential form is,

$$\overline{\nabla} \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

Integrating this eq^n over a surface S, bounded by a curve C we get,

$$\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Converting the surface integral to the L.H.S. into line integral by Stoke's thm, we get,

$$\oint_l \vec{E} \cdot d\vec{l} = \frac{-\partial}{\partial t} \left[\int_S \vec{B} \cdot d\vec{S} \right] \text{ --- } (4)$$

Thus, the emf induced in any conductor is proportional to the rate of change of magnetic flux associated with the conductor.

d] The 4th Maxwell's eq^n is ;

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Integrating this eq^n over a surface S, bounded by curve C, we get,

$$\int_S (\vec{\nabla} \times \vec{H}) \cdot d\vec{S} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

Using Stoke's thm on L.H.S., we get,

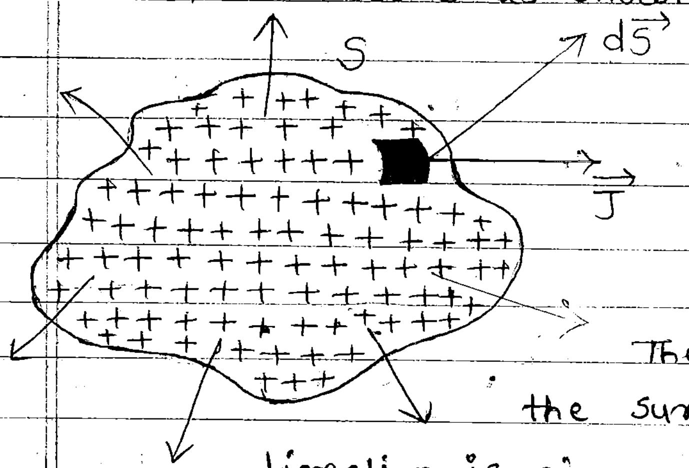
$$\int_S (\vec{\nabla} \times \vec{H}) \cdot d\vec{S} = \oint_l \vec{H} \cdot d\vec{l}$$

$$\therefore \oint_l \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

The magnetomotive force around a closed path is equal to the conduction current plus the time derivative of the electric displacement through any surface bounded by the path.

* **Equation of Continuity :-**

Consider a Conducting region bounded by a closed surface S as shown in the fig.



Let us assume that the volume charge density in the region is ρ. Let $\vec{J}$ be the flux of the Current density vector leaving the surface. The total Current I crossing the surface S in the outward direction is given as,

$$I = \int_S \vec{J}.\vec{dS} \quad \text{——— } (1)$$

If the current density vector $\vec{J}$ remains unchanged, with time everywhere, the current is said to be stationary or steady. Let us consider an infinitesimal small element dv of the region. The charge enclosed by the volume element is

$$dq = \rho\, dv$$

The total charge q inside volume V bounded by the surface S is,

$$q = \int_V \rho\, dv \quad \text{——— } (2)$$

Since, current is simply a flow of charge per unit time, an outward flow of charge from the region will decrease the charge concentration in the bounded region. Thus, from law of conservation of charge, "The rate of charge leaving the volume through bounding surface must be equal to rate of decrease of charge inside it."

$$\therefore \quad I = \frac{-dq}{dt} \quad \text{——} \quad (3)$$

Using eq^n (2) in (3),

$$I = \frac{-d}{dt}\left(\int_V \rho \, dv\right) \quad \text{—} \quad (4)$$

From eq^{n}s (1) & (4) we have,

$$\oint_S \vec{J}.\vec{ds} = \frac{-d}{dt}\left[\int_V \rho \, dV\right] \quad \text{—} \quad (5)$$

Eq^n (5) represents the integral form of the eq^n of Continuity & is mathematical expression of the principle of Conservation of charge. It states that any change of a charge in a given Volume must be accompanied by a flow of charge across the surface bounding the Volume. In other words, charge can neither be created nor be destroyed, it is merely transported.

According to Gauss's divergence thm, the surface integral can be converted into volume integral.

$$\therefore \oint_S \vec{J}.\vec{ds} = \int_V (\vec{\nabla}.\vec{J}) \, dv$$

$$\therefore \int_V (\vec{\nabla}.\vec{J}) \, dv = \frac{-d}{dt}\left(\int_V \rho \, dV\right)$$

Interchanging differentiation & integration on RHS of the above eq^n, we get

$$\int_V (\vec{\nabla}.\vec{J}) \, dv = -\int_V \frac{d\rho}{dt} \, dv \quad \text{—} \quad (6)$$

Since, the volume under consideration is stationary, the differential w.r.t. time in above eq^n can be taken as partial differentiation of volume charge density w.r.t. time

$$\therefore \int_V (\vec{\nabla} \cdot \vec{J})\, dv = -\int_V \frac{\partial \rho}{\partial t}\, dv$$

or

$$\int_V \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} \right) dv = 0$$

As the volume under consideration is arbitrary, the above eq^n can be true only if the integrand in the above eq^n vanishes at every point.

$$\therefore \boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0} \quad \text{——} \quad (7)$$

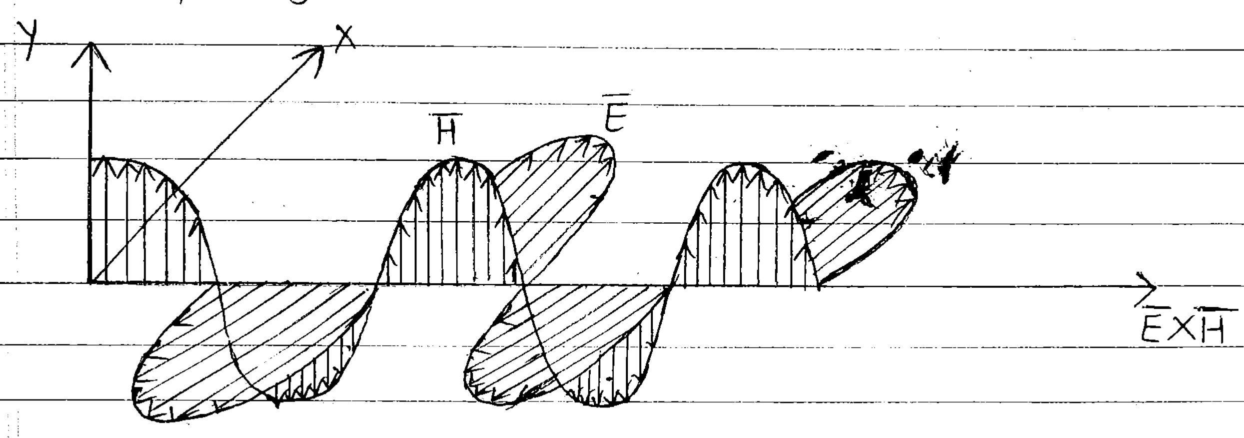
This is called eq^n of continuity.

If there is no net charge within the closed surface then $\rho = 0$

$$\therefore \boxed{\vec{\nabla} \cdot \vec{J} = 0 \quad \text{or} \quad \text{div } J = 0}.$$

* <u>Poynting Vector Theorem</u> :-

we know that a plane electromagnetic wave in free space propagates in the direction of the vector $(\bar{E} \times \bar{H})$.



$\bar{E}$ & $\bar{H}$ are in phase, but perpendicular to each other.

Maxwell's 3rd & 4th relations are :-

$$\left.\begin{aligned} \bar{\nabla} \times \bar{E} &= \frac{-\partial \bar{B}}{\partial t} \\ \bar{\nabla} \times \bar{H} &= \bar{J} + \frac{\partial \bar{D}}{\partial t} = \frac{\partial \bar{D}}{\partial t} \end{aligned}\right\} \quad \text{— (1)} \quad \text{since } \bar{J} = 0.$$

Taking scalar product on both sides of above eq^{n}s with $\bar{H}$ & $\bar{E}$ respectively & subtracting we get,

$$\bar{E} \cdot (\bar{\nabla} \times \bar{H}) - \bar{H} \cdot (\bar{\nabla} \times \bar{E}) = \bar{E} \cdot \frac{\partial \bar{D}}{\partial t} + \bar{H} \frac{\partial \bar{B}}{\partial t}$$

By using vector identity,

$$\bar{\nabla} \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\bar{\nabla} \times \bar{A}) - \bar{A} \cdot (\bar{\nabla} \times \bar{B})$$

$$\therefore \quad \bar{\nabla} \cdot (\bar{E} \times \bar{H}) = \bar{H} \cdot (\bar{\nabla} \times \bar{E}) - \bar{E} \cdot (\bar{\nabla} \times \bar{H})$$

$$= - \left[\bar{E} \cdot (\bar{\nabla} \times \bar{H}) - \bar{H} \cdot (\bar{\nabla} \times \bar{E}) \right]$$

$$= - \left[\bar{E} \cdot \frac{\partial \bar{D}}{\partial t} + \bar{H} \cdot \frac{\partial \bar{B}}{\partial t} \right]$$

Using $\bar{D} = \epsilon_0 \bar{E}$ & $\bar{B} = \mu_0 \bar{H}$ Above eq^n becomes,

$$\bar{\nabla} \cdot (\bar{E} \times \bar{H}) = - \left[\epsilon_0 \frac{\partial E^2}{\partial t} + \mu_0 \frac{\partial H^2}{\partial t} \right]$$

$$= \frac{-1}{2} \left[\frac{\partial}{\partial t} \left(\epsilon_0 E^2 + \mu_0 H^2 \right) \right]$$

$$\therefore \quad \bar{\nabla} \cdot (\bar{E} \times \bar{H}) = \frac{-\partial}{\partial t} \left[\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right] \quad \text{— (A)}$$

Integrating over a volume V bounded by surface S & using divergence thm we get,

$$\int_S (\bar{E} \times \bar{H}) \cdot d\bar{s} = \frac{-\partial}{\partial t} \int_V \left(\frac{\epsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2} \right) dV$$

The quantity on LHS is denoted by $\bar{P}$

$$\bar{P} = \bar{E} \times \bar{H} \quad \text{watts}/m^2.$$

& is called the Poynting vector. It represents the amount of field energy passing through unit area of surface in unit time normal to the direction of flow of energy.

$$\therefore \int_S \bar{P}.d\bar{s} = \frac{-\partial}{\partial t} \int_V \left(\frac{\epsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2} \right) dV$$

The integral on the RHS is the sum of electric & magnetic energies.

Electrostatic field energy per unit volume $= \frac{1}{2} \epsilon_0 E^2$

& Magnetostatic field energy per unit volume $= \frac{1}{2} \mu_0 H^2$ for free space.

RHS is thus the energy lost per unit time by the volume V & LHS must be total outward flux of energy through the surface S bounding volume V. So that $(\bar{E} \times \bar{H})$ has dimensions of energy / (area X time) while the term $-\frac{dW}{dt}$ is the rate at which energy is decreasing within volume where,

$$W = \int_V \left(\frac{\epsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2} \right) dV$$

& is called the total energy (EM energy) in a given volume in free space.

For any arbitrary closed surface, the amount of power flowing out through surface is

$$\oint_S \bar{P}.d\bar{s} = \frac{-dW}{dt}$$

Poynting theorem :- It states that, "Decrease of electromagnetic energy (−ve sign) per unit time in certain volume V is equal to work done (energy) by field forces per unit time plus flux flow outwards per unit time."

* **Wave eq^n for E & H** :-

Whenever EM waves propagated or transmitted in the free space, they reach the destination with the speed equal to the speed of the light.

Recalling Maxwell's Differential eq^{n}s,

$$\bar{\nabla}\cdot\bar{D} = \rho$$

$$\bar{\nabla}\cdot\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

with $\bar{J} = \sigma\bar{E}$, $\bar{B} = \mu_0\bar{H}$, $\bar{D} = \epsilon_0\bar{E}$ —— (1)

& in the free space i.e. vacuum,

$$\rho = 0,\ \epsilon_0 = 1,\ \mu_0 = 1,\ \sigma = 0.$$

So Maxwell's eq^{n}s reduced to,

$$\bar{\nabla}\cdot\bar{E} = 0 \qquad \text{— (A)}$$

$$\bar{\nabla}\cdot\bar{H} = 0 \qquad \text{— (B)}$$

$$\bar{\nabla}\times\bar{E} = -\mu_0\frac{\partial \bar{H}}{\partial t} \qquad \text{— (C)}$$

$$\bar{\nabla}\times\bar{H} = \epsilon_0\frac{\partial \bar{E}}{\partial t} \qquad \text{— (D)}$$

Taking the curl on B.S. of eq^n (C) we have,

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{E}) = -\bar{\nabla}\times\left(\mu_0\frac{\partial \bar{H}}{\partial t}\right)$$

$$= -\mu_0\,\bar{\nabla}\times\frac{\partial \bar{H}}{\partial t}$$

Using the vector identity,

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A}\cdot\bar{C}) - \bar{C}(\bar{A}\cdot\bar{B})$$

$$\therefore \bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = \bar{\nabla}(\bar{\nabla}\cdot\bar{E}) - \bar{E}(\bar{\nabla}\cdot\bar{\nabla})$$

$$= 0 - \nabla^2 \bar{E} \qquad \because \bar{\nabla}\cdot\bar{E} = 0$$

$$\therefore -\nabla^2 \bar{E} = -\mu_0 \bar{\nabla} \times \frac{\partial \bar{H}}{\partial t}$$

since, the Curl operation & the operation of differentiation w.r.t. time t can be interchanged.

$\therefore$ The above eq^n becomes,

$$-\nabla^2 \bar{E} = -\mu_0 \frac{\partial}{\partial t}(\bar{\nabla} \times \bar{H}) \quad \text{— (2)}$$

Substituting from eq^n (D),

$$-\nabla^2 \bar{E} = -\mu_0 \frac{\partial}{\partial t}\left(\epsilon_0 \frac{\partial \bar{E}}{\partial t}\right)$$

$$\therefore -\nabla^2 \bar{E} = -\epsilon_0 \mu_0 \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\therefore \nabla^2 \bar{E} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

i.e. $$\boxed{\nabla^2 \bar{E} - \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} = 0}$$, with $\mu_0 \epsilon_0 = \frac{1}{c^2}$ (*)

Similarly taking the curl on B.S. of eq^n (D), we get,

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{H}) = \bar{\nabla} \times \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$\therefore \bar{\nabla} \times (\bar{\nabla} \times \bar{H}) = \epsilon_0 \bar{\nabla} \times \frac{\partial \bar{E}}{\partial t}$$

Using the vector identity we have,

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{H}) = \bar{\nabla}\cdot(\bar{\nabla}\cdot\bar{H}) - \bar{H}\cdot(\bar{\nabla}\cdot\bar{\nabla})$$

$$= 0 - \nabla^2 \bar{H} \qquad \because \bar{\nabla}\cdot\bar{H} = 0$$

$$\therefore -\nabla^2 \bar{H} = \epsilon_0 \bar{\nabla} \times \frac{\partial \bar{E}}{\partial t}$$

$$\therefore \quad -\nabla^2 \bar{H} = \epsilon_0 \frac{\partial}{\partial t} (\bar{\nabla} \times \bar{E}) \quad \text{——} \quad (4)$$

Substituting from eq^n (C),

$$-\nabla^2 \bar{H} = \epsilon_0 \frac{\partial}{\partial t} \left(-\mu_0 \frac{\partial \bar{H}}{\partial t} \right)$$

$$\therefore \quad -\nabla^2 \bar{H} = -\mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2}$$

$$\therefore \quad \nabla^2 \bar{H} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{H}}{\partial t^2} = 0$$

$$\therefore \quad \boxed{\nabla^2 \bar{H} - \frac{1}{c^2} \frac{\partial^2 \bar{H}}{\partial t^2} = 0}, \quad \text{with } \mu_0 \epsilon_0 = \frac{1}{c^2} \quad \text{——} \quad (**)$$

Eq^{n}s (*) & (**) are known as vector wave eq^{n}s for the field vectors $\bar{E}$ & $\bar{H}$ respectively.

These two eq^{n}s are compared with the standard wave eq^n representing as,

$$\nabla^2 \psi = \frac{1}{V^2} \frac{\partial^2 \psi}{\partial t^2}$$

However, the standard eq^n represents an unattenuated wave travelling at a speed V. So we conclude that field vector $\bar{E}$ & $\bar{H}$ are propagated in free space as waves at a speed

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \text{Velocity of light.}$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m. & $\epsilon_0 = 8.85 \times 10^{-12}$ F/m

Then $C = 3 \times 10^8$ m/s.

* Frequency Range of EM waves & their significance :-

30 Hz		10^7 m
	ELF	
300 Hz		10^6 m
	VF	
3 kHz		10^5 m
	VLF	
30 kHz		10^4 m
	LF	
300 kHz		10^3 m
	MF	
3 MHz		10^2 m
	HF	
30 MHz		10^1 m
	VHF	
300 MHz		1 m
	UHF	
3 GHz		10^{-1} m
	SHF	
30 GHz		10^{-2} m
	EHF	
300 GHz		10^{-3} m
millimeter waves →		
		10^{-4} m
Infrared →		
		0.7×10^{-6} m (Red)
Visible light →		
		0.4×10^{-6} m (Violet)
Ultraviolet →		
X-rays →		
Gamma rays →		
	Cosmic rays	

freq^{n}s ↓

wavelengths ↑

The Electromagnetic spectrum

1] Extremely low frequencies (ELF) :-

The frequency range from 30 Hz to 300 Hz is called as extremely low frequency range. It includes a.c. power line frequencies (60 Hz) & those frequencies in the lower end of the human hearing range.

2] Voice frequencies (VF) :-

The freq^n range from 300 Hz to 3KHZ is called a voice frequency range. This is the normal range of human speech.

3] Very low frequencies (VLF) (3KHz – 30KHz) :-

It includes higher end of the human hearing range. Also used by Government & military communications. These frequencies are used as long distance point to point communications.

4] Low frequencies (LF) (30 KHZ - 300KHZ) :-

They are used in aeronautical & marine navigation.

5] Medium Frequencies (MF) (300 KHZ – 3MHz) :-

These frequencies are used in AM radio broadcasting (535 to 1605 KHZ).

6] High Frequencies (HF) (3MHZ - 30MHz) :-

These are also known as short-wave. Short-wave broadcasting at national & international level takes place in this range.

7] Very high frequencies (VHF) :- (30MHZ – 300 MHZ) :-

This is an extremely popular frequency range & is used by mobile radio, FM radio broadcasting (88 - 108 MHz) & TV channels 2 through 13.

8] Ultrahigh frequencies (UHF) : (300 MHz - 3 GHz) :-
This range of frequencies is extremely used for Communication. This range is used for TV channels 14 through 83, Cellular telephone & military Services.
Frequencies above 1000 MHz = 1 GHz are called microwaves.

9] Super High Frequencies (SHF) (3 GHz - 30 GHz) :-
These are microwave frequencies. Frequencies within the range 1 to 30 GHz are called microwave. They are used in Satellite Communication & RADAR.

10] Extremely high frequencies (EHF) (30 GHz - 300 GHz) :-
This range is not much popular & used in limited activity. It includes Satellite communication & some specialized RADAR.

11] Infrared :- The frequencies upto 300 GHz are called radio waves. Infrared region is between highest radio frequencies & the visible spectrum. They are used in Astronomy to detect Stars & also used in guided missiles, T.V. Remote Control,

12] The visible spectrum :- The visible spectrum is above the infrared region. Also referred to as light. This spectrum is used in fiber optics.

13] Ultraviolet & X-rays :- Used in Investigations

14] Gamma Rays :- Radioactivity.

* Skin Depth :-

A good Conductor is defined as one having a very high Conductivity; Consequently, the Conduction Current is much larger than the displacement Current. The energy transmitted by the wave travelling through the medium will decrease continuously as the wave propagates because Ohmic losses are present. Expressed mathematically, a good Conductor requires the Criterion $\sigma >> \omega\epsilon$

This distance is called as the skin depth. & is denoted by δ.

$$\therefore \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}.$$

At microwave frequencies, the skin depth is extremely short.

* Polarization :- Plane, circular, elliptical :-

- Plane Polarized wave :-

We know that, the EM waves have a electric & magnetic field vectors & they are $\perp^{ar}$ to each other & also always perpendicular to the direction of propagation. If the EM waves are propagating along the +ve Z-dir^n, then $\bar{E}$ & $\bar{H}$ can have components in the Z-dir^n.

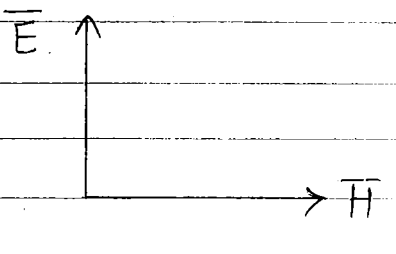
$\therefore$ A plane EM wave propagating in free space has no longitudinal components.

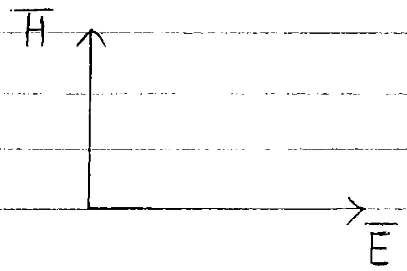
In practice, the EM wave consists of combination of plane waves propagating in the same direction with arbitrary orientations of field vectors, arbitrary magnitudes & random phases is called as an unpolarized wave.

A superposed wave is obtained by the Combination of Various plane waves. The electric field vectors in a Superposed wave or a resultant wave lie in a certain fixed direction, such as a wave is said to be plane polarized wave or linearly polarized wave.

The EM waves having horizontal electric field vector is referred to as horizontally polarized wave.

The EM waves having vertical electric field vector is referred to as vertically Polarized wave.





(a) Vertically Polarized wave (b) Horizontally Polarized wave.

There are 3 types of Polarization :-

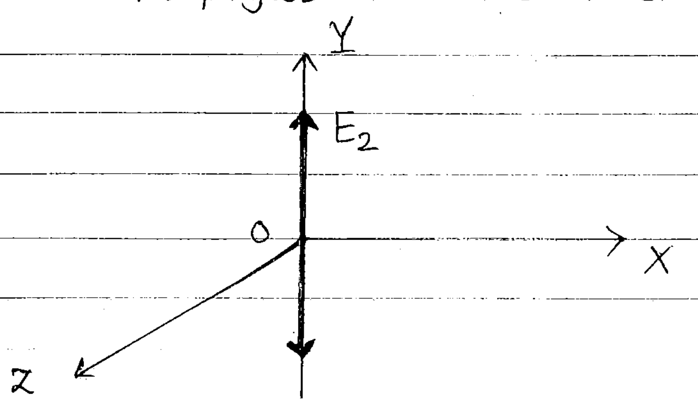
1] Linear or Plane Polarization
2] circular Polarization
3] Elliptical Polarization.

1] Linear or Plane Polarization :-

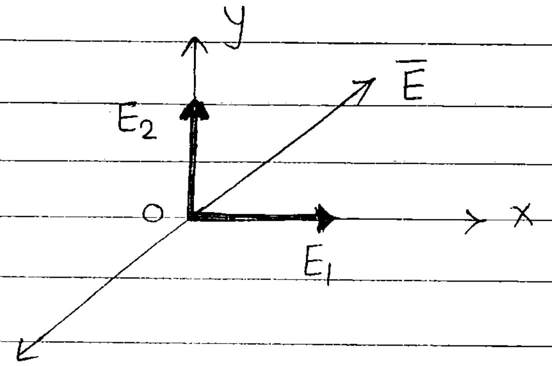
The electric field of linearly polarized wave in Y-dir^n is given by eq^n,

$$E_y = E_2 \sin(\omega t - kz) \quad — \quad (A)$$

This wave Propagates in the +ve z-dir^n.



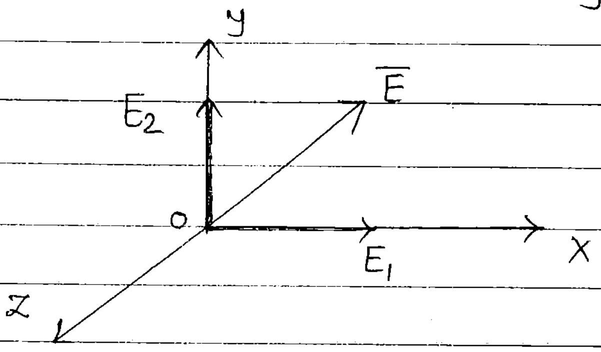
In general, the electric field of wave travelling in the z-dir^n may have both y-component & an x-component as shown in the follow. fig.



Here E_1 is the amplitude of wave which is linearly polarized in the x-dir^n & E_2 is the amplitude of a wave which is linearly polarized in y-dir^n.

2] Elliptical Polarization :-

Consider an EM wave travelling in +ve z-dir^n.



The electric field components in X & Y dir^n are,

$$E_x = E_1 \sin(\omega t - kz) \quad — ①$$

$$E_y = E_2 \sin(\omega t - kz + \delta) \quad — ②$$

where, δ = time phase angle by which E_y leads E_x.

$$\therefore \overline{E} = E_x \hat{i} + E_y \hat{j}$$

$$\therefore \overline{E} = E_1 \sin(\omega t - kz) + E_2 \sin(\omega t - kz + \delta) \quad -③$$

At $z = 0$,

$$E_x = E_1 \sin \omega t \quad — ④$$

$$E_y = E_2 \sin(\omega t + \delta) \quad — ⑤$$

$$\therefore E_y = E_2\left[\sin\omega t \cos\delta + \cos\omega t \sin\delta\right] \quad \text{--- (6)}$$

From eq^n (4),

$$\sin\omega t = E_x / E_1$$

Also, $\cos\omega t = \sqrt{1-\sin^2\omega t}$

$$\therefore \cos\omega t = \sqrt{1 - \frac{E_x^2}{E_1^2}}$$

$\therefore$ Substituting in eq^n (6) we get,

$$E_y = E_2\left[\frac{E_x}{E_1}\cos\delta + \sqrt{1-\frac{E_x^2}{E_1^2}}\,\sin\delta\right] \quad \text{--- (7)}$$

$$\therefore E_y - \frac{E_2}{E_1}E_x\cos\delta = E_2\sqrt{1-\frac{E_x^2}{E_1^2}}\,\sin\delta$$

Squarring B.S.,

$$E_y^2 - 2\frac{E_2}{E_1}E_xE_y\cos\delta + \frac{E_2^2}{E_1^2}E_x^2\cos^2\delta$$

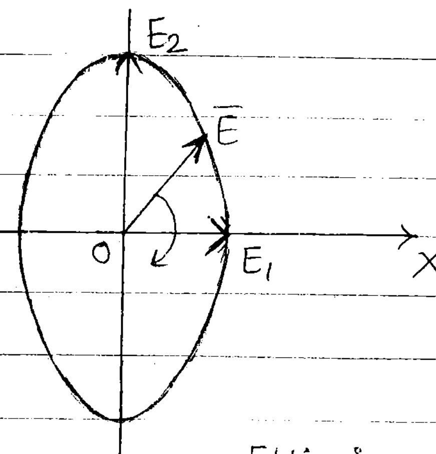
$$= E_2^2\left(1-\frac{E_x^2}{E_1^2}\right)\sin^2\delta$$

$$\therefore E_y^2 + \frac{E_x^2}{E_1^2}E_2^2 - 2\frac{E_2E_xE_y}{E_1}\cos\delta = E_2^2\sin^2\delta \quad \text{--- (8)}$$

Dividing eq^n (8) by E_2^2 we get,

$$\frac{E_y^2}{E_2^2} + \frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1}\cos\delta = \sin^2\delta \quad \text{--- (9)}$$

This is the general expression of an ellipse. It represents elliptically polarized wave.



Elliptical Polarization

3] circular Polarization :-

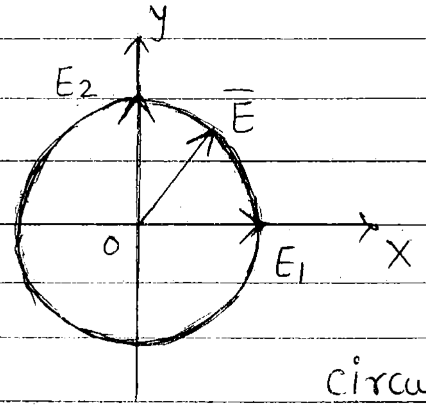
$$\frac{E_y^2}{E_2^2} + \frac{E_x^2}{E_1^2} - \frac{2 E_x E_y}{E_1 E_2} \cos\delta = \sin^2\delta \quad \text{— (1)}$$

If $E_1 = E_2$ & $\delta = \frac{\pi}{2}$ then eq^n (1) becomes circularly polarized wave.

$$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1 \qquad \left(\because \cos\frac{\pi}{2} = 0\right)$$

$$\therefore \quad E_x^2 + E_y^2 = E_1^2 \quad \text{— (2)}$$

This is the eq^n of circle of radius E_1. eq^n (2) shows that the magnitude of E remains constant although its dir^n rotates.



Circular Polarization

When $\delta = +\pi/2$, the wave is left circularly polarized & when $\delta = -\pi/2$, the wave is right circularly polarized.

* Reflection & Refraction of EM waves from boundary of two dielectrics :-

Boundary Conditions :-

a] The normal component of a magnetic induction is continuous across boundary i.e. $B_{n_1} = B_{n_2}$

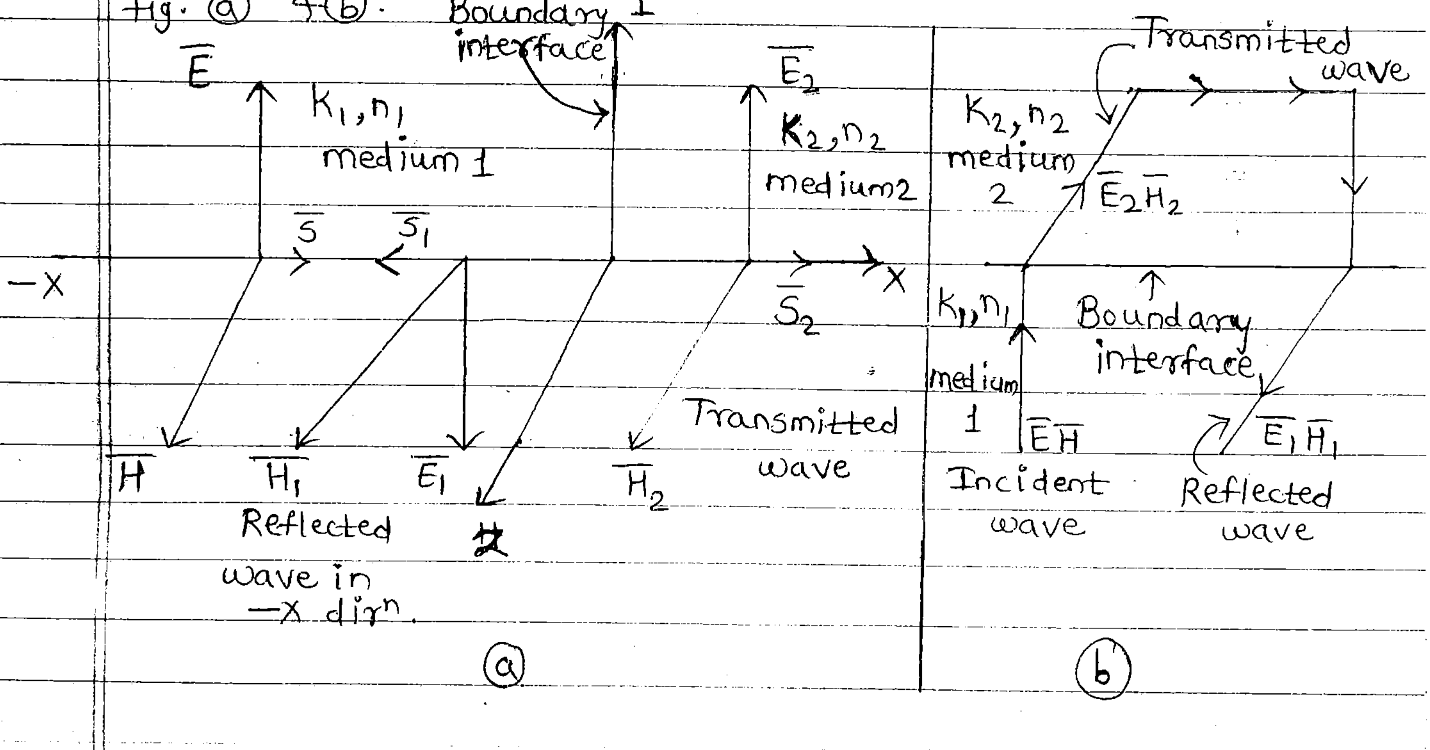
b] The tangential component of electric field $\vec{E}$ is continuous across the interface i.e. $E_{t_1} = E_{t_2}$

c] The normal component of a electric displacement $\vec{D}$ is continuous across the interface i.e. $D_{n1} = D_{n2}$.

d] The tangential component of magnetic intensity $\vec{H}$ is continuous across the surface separating two dielectrics i.e. $H_{t_1} = H_{t_2}$.

A plane EM wave is defined as a wave whose phase is the same at a given instant at all the points, in each plane $\perp^{ar}$ to the some specified direction. Suppose if this dir^n is along Z-axis, then E must have the same phase at all the points that have the same Z-value.

According to Snell's law, we know that when EM waves propagating in material usually enter the material through a boundary bet^n it having the two medium with different dielectric constants K_1 & K_2. Let us consider an EM wave which is incident normally on a dielectric boundary interface. In this case, some of the incident energy is reflected & remaining is transmitted into the dielectric as shown in fig. ⓐ & ⓑ.



From the above fig ⓐ & ⓑ, $\overline{E}$ & $\overline{H}$ describe the incident wave travelling in the +ve X-dir^n. $\overline{E_1}$ & $\overline{H_1}$ describe the reflected wave travelling in the -ve X-dir^n while $\overline{E_2}$ & $\overline{H_2}$ are the transmitted rays & boundary interface is taken along YZ plane. With the medium 1 on the LHS & medium 2 on the RHS having the dielectric Constants K_1 & K_2 also having unit vectors $\overline{n_1}$ & $\overline{n_2}$.

If these two media are isotropic, homogeneous, Stationary, charge free, linear, non-Conducting & infinitely extended, then the electric & magnetic field vectors of reflected & transmitted waves are obtained by using the Boundary Condition & by applying the Right hand rule to the cross product of $\overline{P} = \overline{E} \times \overline{H}$.

The propagations of EM waves in a matter for non-Conducting medium, therefore the refractive index n is equal to the Square root of the dielectric Constant of the medium which is given as,

$$n = \sqrt{K} \quad \left\{ \begin{array}{l} \text{we know that, } c^2 = \mu_0 \epsilon_0 ,\ V^2 = \mu\epsilon \\ \therefore n = V/c = \sqrt{\mu\epsilon} / \sqrt{\mu_0 \epsilon_0} = \epsilon/\epsilon_0 \end{array} \right.$$

where K = dielectric constant $= \epsilon/\epsilon_0$

where ϵ = Permitivity of the medium &

ϵ_0 = Permitivity of the free space.

$$\therefore n = \sqrt{\epsilon/\epsilon_0} \quad \text{——} \quad (1)$$

The relation between magnetic field induction $\overline{B}$ & electric field $\overline{E}$ is given by,

$$B = \sqrt{\mu\epsilon}\ E \qquad \text{But } B = \mu H.$$

$$\therefore \mu H = \sqrt{\mu\epsilon}\ E$$

$$\therefore H = \frac{\sqrt{\mu\epsilon}}{\mu} E = \sqrt{\frac{\mu\epsilon}{\mu^2}}\ E$$

$$H = \sqrt{\frac{\epsilon}{\mu}}\ E$$

But $\epsilon = \epsilon_0$, $\mu = \mu_0$

$$\therefore \quad H = n_1 \sqrt{\frac{\epsilon_0}{\mu_0}}\, E \quad \text{—— (2)}$$

This eq^n will represent incident wave eq^n. Similarly, the reflected wave eq^n can be written as,

$$H_1 = n_1 \sqrt{\frac{\epsilon_0}{\mu_0}}\, E_1 \quad \text{—— (3)}$$

similarly, the eq^n for transmitted wave,

$$H_2 = n_2 \sqrt{\frac{\epsilon_0}{\mu_0}}\, E_2 \quad \text{—— (4)}$$

Since we are considering normal incidence of EM wave, boundary conditions for D_n & B_n are meaningless & it themselves are zero.

$\therefore$ By applying the boundary condition as tangential componen only $\overline{E}$ & $\overline{H}$,

$$\therefore \quad E = E_1 + E_2$$

$$\therefore \quad E - E_1 = E_2 \quad \text{—— (5)}$$

similarly, $\quad H + H_1 = H_2 \quad$ —— (6)

Substituting the values of H, H_1 & H_2 in eq^n (6) from the eq^n (2), (3) & (4) we get,

$$n_1 \sqrt{\frac{\epsilon_0}{\mu_0}}\, E + n_1 \sqrt{\frac{\epsilon_0}{\mu_0}}\, E_1 = n_2 \sqrt{\frac{\epsilon_0}{\mu_0}}\, E_2$$

$$\therefore \quad \cancel{\sqrt{\frac{\epsilon_0}{\mu_0}}} \left(n_1 E + n_1 E_1 \right) = \cancel{\sqrt{\frac{\epsilon_0}{\mu_0}}}\, n_2 E_2$$

$$\therefore \quad n_1 E + n_1 E_1 = n_2 E_2 \quad \text{—— (7)}$$

To find the value of E_2, multiply by n_1 to eq^n (5) we get,

$$n_1 E - n_1 E_1 = n_1 E_2 \quad \text{—— (8)}$$

Adding eq^{n}s (7) & (8) we get,

$$\begin{array}{rl} & n_1 E + \cancel{n_1 E_1} = n_2 E_2 \\ + & n_1 E - \cancel{n_1 E_1} = n_1 E_2 \\ \hline & 2 n_1 E = (n_1 + n_2) E_2 \end{array}$$

$$\therefore \boxed{E_2 = \frac{2n_1 E}{n_1 + n_2}} \quad \text{—— (9)}$$

To find the value of E_1, substituting eq^n (9) in (5) we get,

$$E - E_1 = E_2$$

$$\therefore \quad E - E_1 = \frac{2n_1 E}{n_1 + n_2}$$

$$2n_1 E = \frac{n_1 E + n_2 E - n_2 E_1 - n_1 E_1}{1}$$

$$\therefore \quad -n_1 E + n_2 E - n_2 E_1 - n_1 E_1 = 0$$

$$\therefore \quad E(n_2 - n_1) - E_1(n_1 + n_2) = 0$$

$$\therefore \quad \boxed{E_1 = \frac{(n_2 - n_1) E}{n_1 + n_2}} \quad \text{—— (10)}$$

In similar way, the values of H_1 & H_2 are,

$$\left.\begin{aligned} H_1 &= \frac{(n_2 - n_1) H}{n_1 + n_2} \quad \& \\ H_2 &= \frac{2n_2 \, H}{n_1 + n_2} \end{aligned}\right\} \quad \text{—— (11)}$$

The flux of the EM energy wave per unit area

$$= \bar{P} = \bar{E} \times \bar{H}$$

$$\therefore \quad |\bar{P}| = |\bar{E} \times \bar{H}|$$

$$\therefore \quad |\bar{P}| = EH \sin 90^\circ \qquad \because \sin 90^\circ = 1$$

$$\therefore \quad \bar{P} = EH$$

$$\& \quad EH = E_1 H_1 + E_2 H_2$$

- Reflection & Transmission Coefficient :-

The reflection & transmission coefficients are related to the flow of energy across the interface. The reflection coefficient R_n at the interface bet^n two non-conducting media is defined as,

" R_n is the ratio of magnitude of reflected energy flux per unit area per sec at the interface to the magnitude of incident energy flux per unit area per sec at interface.

OR $R_n = \dfrac{\text{reflected energy}}{\text{Incident energy}}$

$$\therefore R_n = \frac{E_1 H_1}{EH}$$

$$\therefore R_n = \frac{\dfrac{(n_2-n_1)\cancel{E}}{n_1+n_2} \quad \dfrac{(n_2-n_1)\cancel{H}}{n_1+n_2}}{\cancel{E}\cancel{H}}$$

$$\boxed{R_n = \left(\frac{n_2-n_1}{n_1+n_2}\right)^2}$$

- Transmission Coefficient :-

Transmission coefficient at the interface betn two non-Conducting media is defined as T_n.

"T_n is the ratio of magnitude of transmitted energy flux per unit area per sec at interface to the incident energy flux per unit area per sec at interface".

OR $T_n = \dfrac{\text{Transmitted energy}}{\text{Incident energy}}$

$$= \frac{E_2 H_2}{EH}$$

$$= \frac{\dfrac{2n_1\cancel{E}}{n_1+n_2} \cdot \dfrac{2n_2\cancel{H}}{n_1+n_2}}{\cancel{E}\cancel{H}}$$

$$\boxed{T_n = \frac{4n_1 n_2}{(n_1+n_2)^2}}$$

- ## Relation betⁿ T_n & R_n :-

Show that $R_n + T_n = 1$.

Proof :- L.H.S

$$= R_n + T_n$$

$$= \frac{(n_2 - n_1)^2}{(n_1+n_2)^2} + \frac{4n_1 n_2}{(n_1+n_2)^2}$$

$$= \frac{n_2^2 - 2n_1n_2 + n_1^2 + 4n_1n_2}{(n_1+n_2)^2}$$

$$= \frac{n_1^2 + 2n_1n_2 + n_2^2}{(n_1+n_2)^2}$$

$$= \frac{(n_1+n_2)^2}{(n_1+n_2)^2}$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

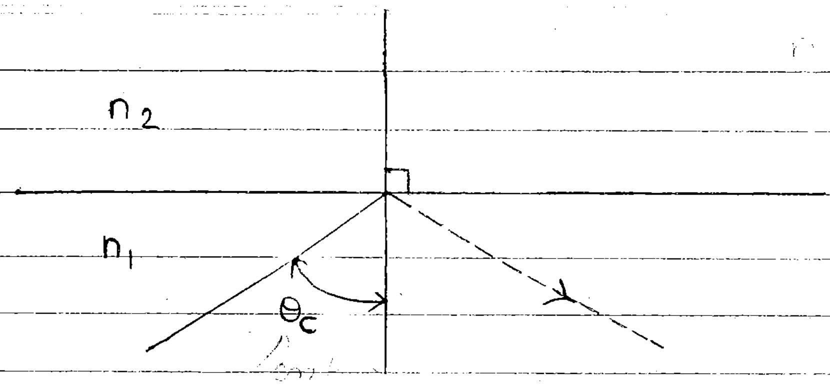
- ## Critical angle :-

If the refractive index of the medium 1 (n_1) is greater than R.I. of the medium 2 (n_2), the angle of refraction is always greater than the angle of incidence. Thus when the angle of refraction is 90° & the refracted ray emerges parallel to the interface betⁿ the dielectrics, the angle of incidence must be less than 90°. This is the limiting case of refraction & the angle of incidence is known as the critical angle θ_c.

The value of the critical angle θ_c is,

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}}$$

At angles of incidence greater than the critical angle, the light is reflected back into the originating medium (total internal reflection).



- **Brewster angle :-**

If the refractive index of the medium 1 (n_1) is equal to the refractive index of the medium 2 (n_2), that is ~~the~~ if the two media are optically indistinguishable. or if $(\theta_1 + \theta_3) = \frac{\pi}{2}$, then $\tan(\theta_1+\theta_3) = \infty$ & amplitude of the reflected wave is zero. In this case, the two media are optically indistinguishable since, $E_2 = 0$. The incident light (unpolarized) has been completely polarized.

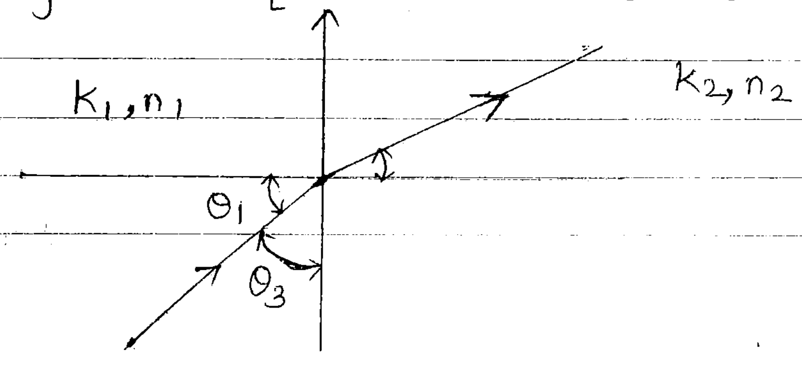
$$\therefore \; n_1 \sin\theta_1 = n_2 \sin\theta_3$$

& substituting, $\theta_3 = \frac{\pi}{2} - \theta_1$ we get,

$$\frac{\sin\theta_1}{\sin\theta_3} = \frac{\sin\theta_1}{\sin\left(\frac{\pi}{2}-\theta_1\right)} = \frac{n_2}{n_1}$$

$$\text{or} \quad \boxed{\tan\theta_1 = \frac{n_2}{n_1}} \qquad \because \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

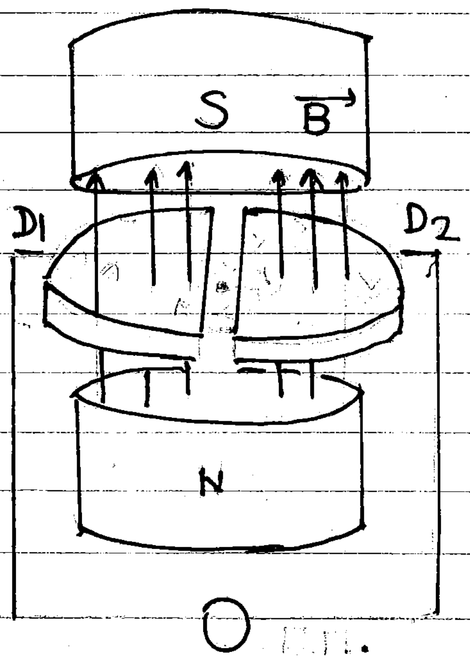
The quantity θ_1 is known as Brewster angle. The relationship betn θ_1 & the refractive indices as given by above eqn is known as Brewster Law.



* Cyclotron :- operating principle :-

When the electric field E & the magnetic field flux density B are at right angles to each other, a magnetic force is exerted onto the e$^{\ominus}$ beam. This type of field is called as Crossed field.

In a crossed field, e$^{\ominus}$s emitted by cathode are accelerated by the electric field & gain velocity, but greater their velocity, the more their path is bent by the magnetic field. The device which operates on this principle is called as Cyclotron.



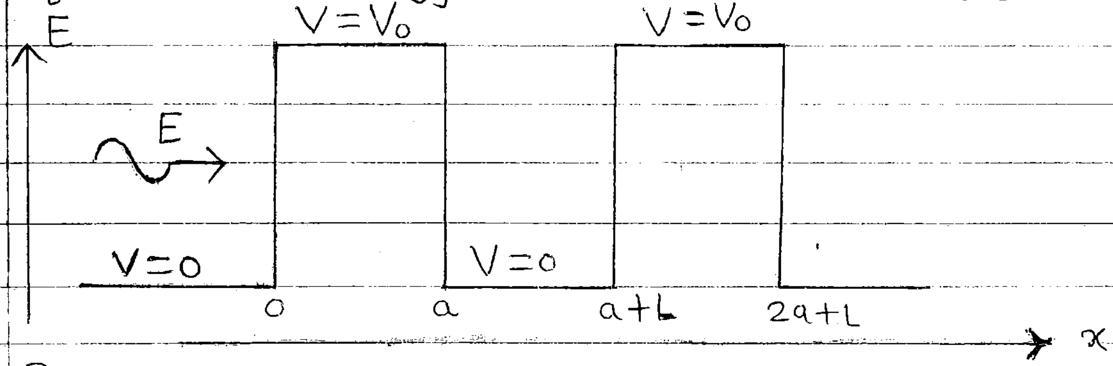
Ch. IV

Applications

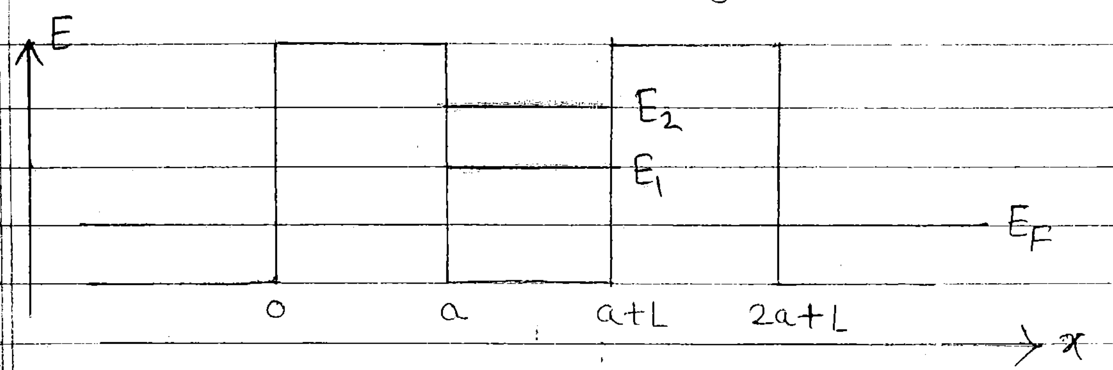
Nanostructure Devices

1] Resonant - tunneling diode :-

Consider two barriers separated by a small distance forming a potential well as shown in the follow. fig (a). We assume that the barriers are sufficiently thin to allow tunneling & that the well region between the two barriers is also sufficiently narrow to form discrete (quasi-bound) energy levels as shown in fig (b).



(a) Double barrier system forming a potential well.



(b) Quantized energy levels in the well.

The quantized energy levels in the double barrier system potential well are given by,

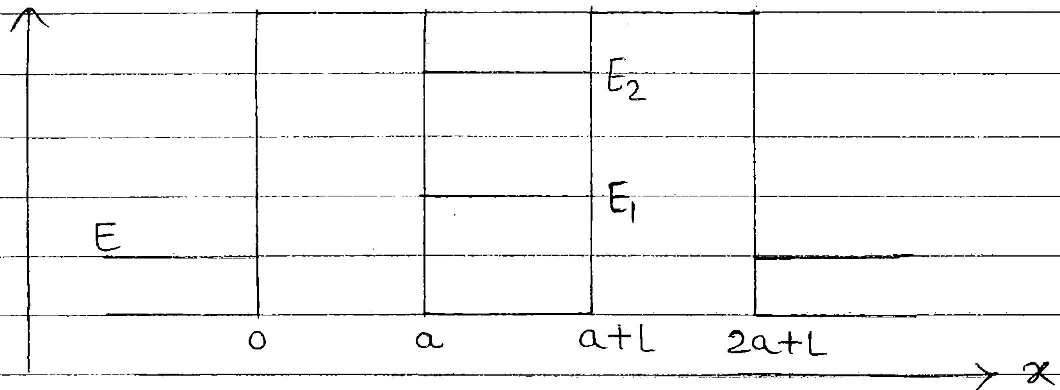
$$E = E_n = \frac{\hbar^2}{2m_e^*}\left(\frac{n\pi}{L}\right)^2$$

which is exactly the same as the result of the quantized energy levels in a one-dimensional quantum well.

The double barrier tunnel junction has important applications to a device known as a resonant tunneling diode. The operation of these diodes can be appreciated from considering the effect of bias on the energy band diagrams for the double barrier system.

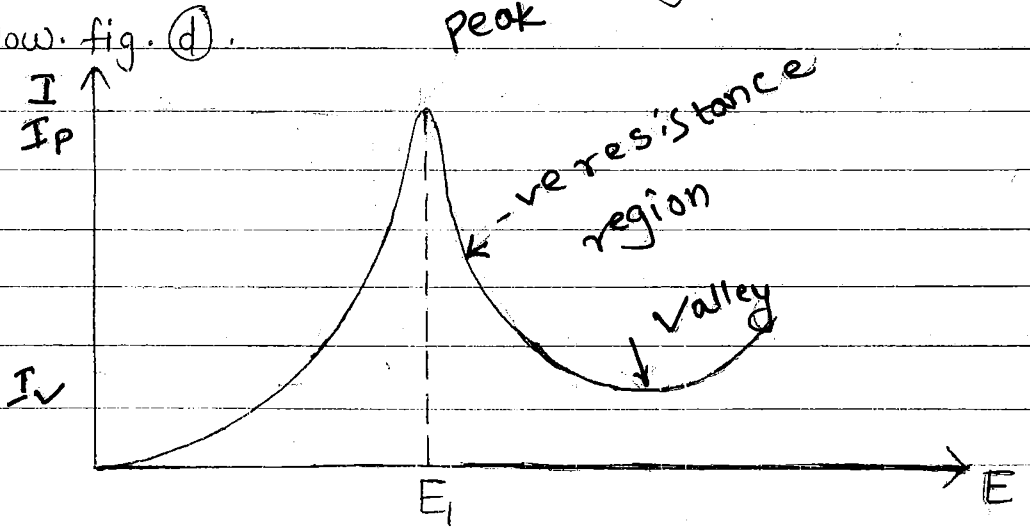
When the incident energy E is very different from the energy of a quasi-bound state E_n, the transmission will be low. & as an $E \to E_n$, transmission will increase. becoming a maximum when $E = E_n$.

E.g. Assume that incident electrons have energy E, & that at first, all of the quasi-bound states E_n lie above E as shown in fig. (c).



(c): Double barrier junction with no applied bias.

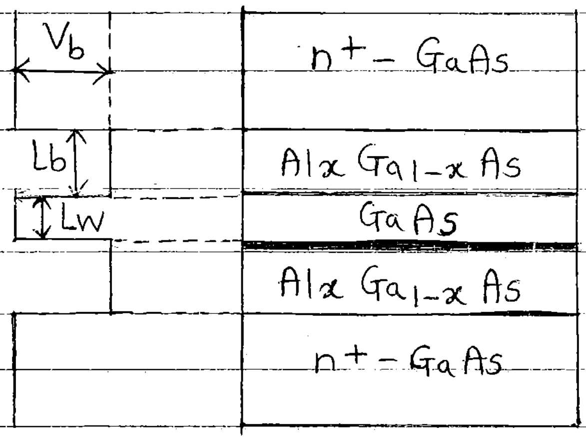
As E increases, tunneling will increase, reaching a peak when $E = E_1$. After that point, a further increase in E will result in a decreasing current as shown in the follow. fig. (d).



ⓓ :- Current – energy characteristic for resonant tunneling Junction

This decrease of current with an increase of bias is called negative resistance. Further peaks & valleys will occur as E approaches & then moves past, other quasi-bound states.

Tunneling is controlled by applying a bias voltage across the device. For the case of no applied bias, the energy band diagram is similar to the shown in fig. ⓒ.



ⓓ Structure of double barrier resonant-tunneling junction

A typical structure is made by using n-type GaAs for the regions to the left & right of the both barriers intrinsic GaAs for the well region & AlGaAs or AlAs for the barrier material.

✱ Electrons in Quantum Wells, Wires & Dots :-

Assume that an electron reside in a 3-dimensional region of space as shown in the fig.

If $\lambda_e << L_x, L_y, L_z$ then the electrons will be free in all directions (i.e. they will act like free particles), & we have an effectively 3-dimensional system. i.e. the system in all directions is large compared with the size scale of the electrons.

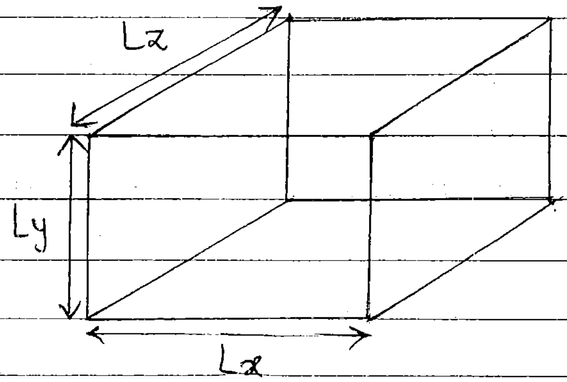


fig: 3-dimensional region of space : $L_{xyz} \gg \lambda_e$

Even though the space is finite & therefore the possible energy levels of the electron are discrete, because the space is relatively large, the discrete energy levels form essentially a ~~quantum~~ Continuum.

e.g. If we assume that the boundaries of the space have hard walls, then the energy levels are given by,

$$E = \frac{\pi^2 \hbar^2}{2m_e}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right] \quad \text{— (1)}$$

since, L_{xyz} are very large & n_{xyz} are integers, the ratios n_x/L_x, n_y/L_y & n_z/L_z vary almost continuously from a very small values to very large values. Making the replacements,

$$\frac{\pi n_x}{L_x} \rightarrow k_x, \quad \frac{\pi n_y}{L_y} \rightarrow k_y, \quad \frac{\pi n_z}{L_z} \rightarrow k_z \quad \text{— (2)}$$

where, k_x, k_y & k_z are continuous variables we have,

$$E = \frac{\hbar^2}{2m_e}\left(k_x^2 + k_y^2 + k_z^2\right) \quad \text{— (3)}$$

$$= E_{Cont}\left(k_x, k_y, k_z\right) \quad \text{— (4)}$$

If L_{xyz} are large but yet finite, $E_{cont}(k_x, k_y, k_z)$ is an approximately Continuous energy profile & as $L_{xyz} \to \infty$, $E_{cont}(k_x, k_y, k_z)$ becomes the purely Continuous energy for an electron in an infinite space.

The de Broglie wavelength gives the "size" of the $e^{\ominus}$ i.e. a region of space having length L is "Large" if $L \gg \lambda_e$ & "small" if $L \lesssim \lambda_e$. While this is true, the de Broglie wavelength depends on the energy of electron & Fermi energy. The de Broglie wavelength at the Fermi energy is called the fermi wavelength & is denoted by the symbol λ_F.

$\therefore$ For a space to be sufficiently "large" so that the energy levels of the electron form an approximately Continuous set, we usually require $L_x, L_y, L_z \gg \lambda_F$.

1] Quantum Wells :-

Assume that we make the space narrow in one direction means,

$$L_x \lesssim \lambda_F \ll L_y, L_z.$$

In this case we can write as,

$$E = \frac{\hbar^2 \pi^2}{2m_e}\left(\frac{n_x}{L_x}\right)^2 + \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right] \quad \text{①}$$

$$\therefore E = \frac{\hbar^2\pi^2}{2m_e}\left(\frac{n_x}{L_x}\right)^2 + \frac{\hbar^2\pi^2}{2m_e}\left(k_y^2 + k_z^2\right) \quad \text{—②}$$

$$= E_{nx} + E_{cont}(k_y, k_z) \quad \text{—③}$$

Where, since L_x is relatively small, E_{nx} represents discrete one-dimensional energies. Since L_y & L_z are relatively large, two of the substitutions in eqn ② leads to $E_{cont}(k_y, k_z)$ which represents an approximately Continuous energy profile. In this case, electron movement will be Confined in the X-dirⁿ (i.e. $e^{\ominus}$s will feel the boundary in the X-dirⁿ), exhibiting energy quantization in that direction & will be free in the two other direction. This makes for an effectively two-dimensional system called a two-dimensional electron gas, also called a quantum well. The discrete energy levels given by n_x form the subbands.

An $e^{\ominus}$ is in a certain discrete energy level n_x, i.e. it is in a certain subband, but it is otherwise free in the y-z plane.

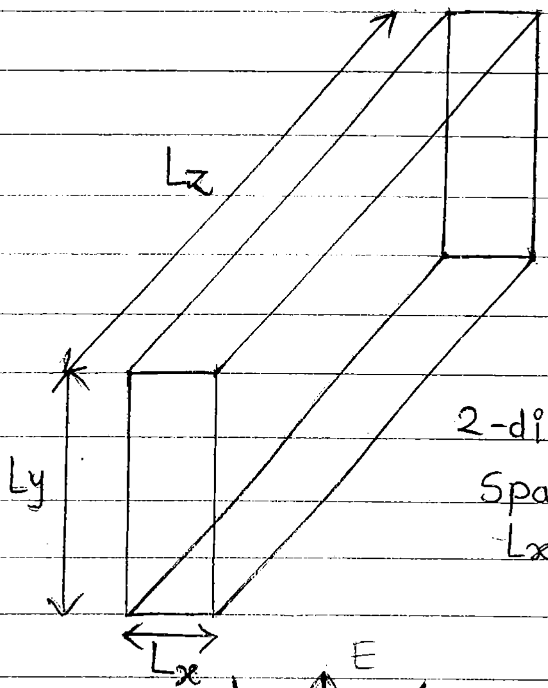
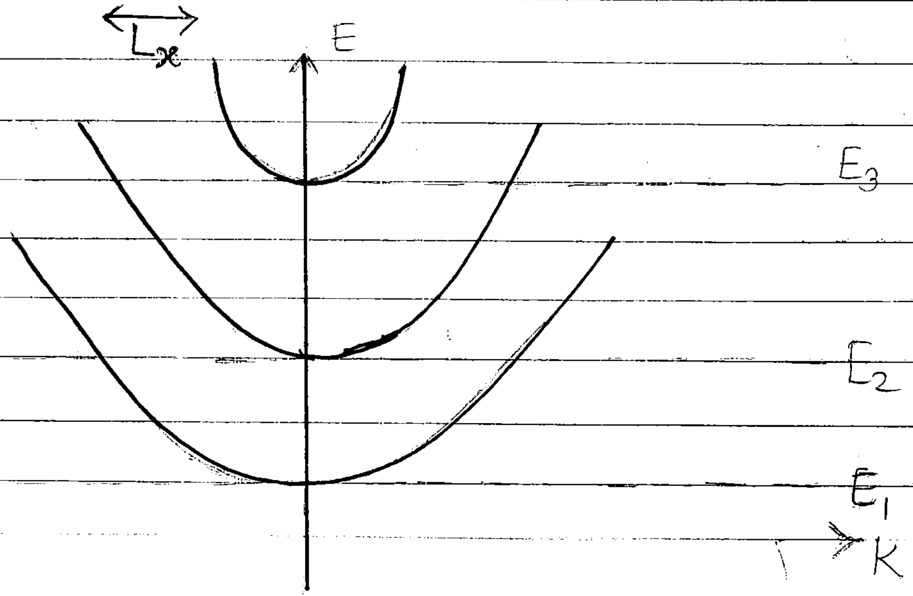


Fig:- A Quantum Well.

2-dimensional region of Space, where $L_x \leq \lambda_F << L_y, L_z$.



To obtain the wave function we can consider the electron to be totally free in the y-z plane, but constrained in the x-dir^n by hard walls at $x=0$ & $x = Lx$.

Schrodinger's eq^n with $V=0$ is,

$$\nabla^2 \psi(r) + \frac{2m}{\hbar^2} E\psi(r) = 0$$

$$\therefore \quad \frac{-\hbar^2}{2m_e} \nabla^2 \psi(r) = E\psi(r) \qquad \text{--- (4)}$$

The wavefunction in product form,

$$\psi(x,y,z) = \psi_x(x)\, e^{ik_y y}\, e^{ik_z z} \qquad \text{--- (5)}$$

where the plane-wave factor contains the continuous wavevectors k_y & k_z. The boundary conditions

$$\psi_x(0) = \psi_x(Lx) = 0$$

lead to

$$\psi(x,y,z) = \left(\frac{2}{Lx}\right)^{1/2} \sin\frac{n_x \pi}{Lx} x\, e^{ik_y y}\, e^{ik_z z} \qquad \text{--- (6)}$$

where, $n_x = 1, 2, 3 \ldots$ & where the allowed energy is given by eq^n (2).

2] **Quantum Wires :-**

Assume that we make the original space shown in the following fig., narrow in two directions

$$Lx, Ly \le \lambda_F << L_z$$

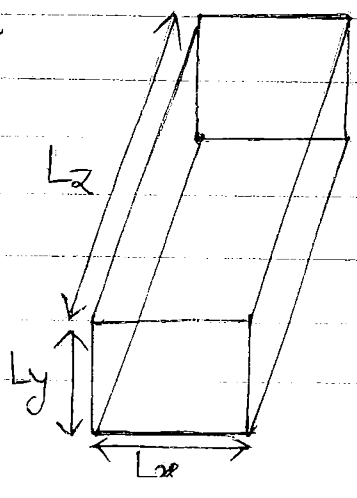


Fig :- a quantum ~~dot~~ wire, one-dimensional space.

$Lx,y \le \lambda_F$

$Lz >> \lambda_F$

$$E = \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2\right] + \frac{\hbar^2\pi^2}{2m_e}\left(\frac{n_z}{L_z}\right)^2 \quad \text{— (1)}$$

$$= \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2\right] + \frac{\hbar^2\pi^2 k_z^2}{2m_e} \quad \text{— (2)}$$

$$= E_{n_x,n_y} + E_{cont}(k_z) \quad \text{— (3)}$$

where, since L_x & L_y are both relatively small, E_{n_x}, E_{n_y} represents discrete, 2 dimensional subband energies & since L_z is relatively large, $E_{cont}(k_z)$ represents an approximately continuous energy profile. In this case, $e^{\ominus}$ movement will be confined in the x-y plane (i.e. $e^{\ominus}$s will feel boundaries in the x & y-dir^{n}s), exhibiting energy quantization in that plane & will be ~~free~~ free in the z-dir^n. This makes for an effectively one-dimensional system called a quantum wire.

Regarding the $e^{\ominus}$ as being totally free, in one dir^n, but constrained by hard walls at $x=0$, L_x & $y=0$, L_y.

Schrodinger's eq^n with $V=0$,

$$\nabla^2\psi(r) + \frac{2m_e}{\hbar^2}E\psi(r) = 0$$

$$\therefore \; -\frac{\hbar^2}{2m_e}\nabla^2\psi(r) = E\psi(r) \quad \text{— (4)}$$

will have solutions in the product form,

$$\psi(x,y,z) = \psi_x(x)\,\psi_y(y)\exp(ik_z z)$$

Using the boundary conditions,

$$\left.\begin{aligned}\psi_x(0) &= \psi_x(L_x) = 0\\ \psi_y(0) &= \psi_y(L_y) = 0\end{aligned}\right\} \quad (6)$$

we obtain,

$$\Psi(x,y,z) = \left(\frac{4}{L_x L_y}\right)^{1/2} \sin\frac{n_x\pi}{L_x}x \,\sin\frac{n_y\pi}{L_y}y\, e^{ik_z z} \quad \text{--- (7)}$$

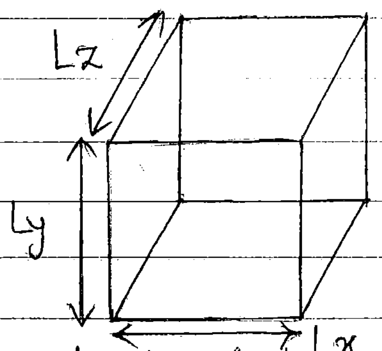
where, $n_{x,y} = 1, 2, \ldots$ & where energy is given by eq^n (2).

3] Quantum dots :-

Now assume that we make the space small in all 3 dir^ns, $L_x, L_y, L_z \le \lambda_F$ as shown in the fig. In this case, we can write as,

$$E = \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right] \quad \text{--- (1)}$$

$$= E_{n_x,n_y,n_z} \quad \text{--- (2)}$$



where since L_x, L_y & L_z are all relatively small, E_{n_x,n_y,n_z} represents discrete energies. $e^\ominus$ movement will be confined in the all 3 dir^ns. (i.e. $e^\ominus$s will feel the boundaries in the x, y & z dir^ns), exhibiting energy quantization in 3 dimensions & will not be free in any dir^n. This makes for an effectively zero-dimensional system called quantum dot.

Since the typical $e^\ominus$ of interest has a fermi wavelength on the order of nanometer in metals, & many tens of nm in semiconductors, quantum dots are nanoscale pieces of material, typically ranging in size from several nm to hundreds of nm.

∴ Quantum dot Contains from several hundred to several hundred thousand atoms.

Quantum dots are typically much larger than atoms, but are generally too small to act like a bulk solid. An $e^{\ominus}$ in a quantum dot will act more like an $e^{\ominus}$ in molecule than $e^{\ominus}$ in a bulk solid, & for this reason, quantum dots are sometimes called artificial molecules.

* Flash Memory :-

It is a non-volatile computer storage chip that can be electrically erased & reprogrammed.
It was developed from EEPRom (electrically erasable Programmable Rom) & must be erased in fairly large blocks before these can be rewritten with new data.

– characteristics :-

1] It is a non-volatile memory.
2] It stores information in array of memory Cells made from floating gate transistors.
3] In flash memory, each memory cell resembles a standard MOSFET.